# A portrait of the mathematical tribe

Marco LiCalzi

## 1 Introduction

In 2009, the photographer Marianna Cook published a fascinating collection of ninety-two photographic portraits of mathematicians [1]; see also [2]. Her single-page preface to the book exudes the same insight and sensibility that animate her pictures. The first sentence, presumably written after having met and photographed so many of them, claims that "mathematicians [...] may look like the rest of us, but they are not the same." If the external appearance is the same, the differences must be somewhere else: here we sketch a (verbal) portrait of what makes them a distinct tribe.

There are of course brilliant epigrams that zero in on a single difference at a time. For instance, Godfrey H. Hardy[1] (1877–1947) writes in *A Mathematician's Apology* that "a mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*." But an honest portrait should muster more, and pay attention to the culture, the mores, and the activities of the community.

Our canvas begins with the anthropological viewpoint. The online *Encyclopædia Britannica* states that a tribe is "a notional form of human social organization based on a set of smaller groups (known as bands), having temporary or permanent political integration, and defined by traditions of common descent, language, culture, and ideology." Wishing to expand our color palette, we combed a few more dictionaries and crafted our portrait of the mathematical tribe as a group whose members are aware of their common identity and keep track of their ancestry (Section 2), speak the same language (Section 3), and share the same culture (Section 4). This tribe permanently occupies (or traverse) many scientific territories, claiming rights that are recognized by its neighbors and often discovering whole new regions.

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[1] When the named person is a mathematician, we add life dates.

## 2 Identity

### Origins

The most ancient mathematical texts are from Mesopotamia and Egypt. Ancient Babilonia also appears in the Tower of Babel narrative (Genesis 11:1–9) about the confusion of tongues, as well as in the related mythical account of *Enmerkar and the Lord of Aratta* where Enmerkar invents writing on clay tablets.

Babylon (called Shinar in the Bible) was probably the largest city of its times. As the capital of a vast empire, it was visited by many delegations of people who spoke different languages. It is conjectured that the Hebrews might have incorrectly associated the name of the city (in akkadian: God's Gate) with the Hebraic noun *bālal* (confusion). The association between the linguistic chaos of the capital and the towering size of its (then) unfinished ziggurat might be at the origin of the narrative.

The very same Babylon is widely recognized as the cradle of three important inventions: alphabet, mathematics, and written law. Words, numbers and law are the hallmark of civilization.

The connection between words and numbers was prominent among the ancient Greeks, who denoted numerals using letters of the alphabet. Even today, many European languages echo this ancient connection across resemblances between their verbs: to count versus to recount (English); compter/conter (French); zahlen/erzhalen (German); or contare/raccontare (Italian). More subtly, the ancient Greek conceived the meaning of *logos* as encompassing the words of the discourse, the reasoning implicit in their use, and the order ruling the cosmos (as opposed to chaos). Nowadays, the word *rational* exemplifies a similar accretion of meanings.

Against this background, the school founded by Pythagoras (6th century BC) in Croton is usually considered the first community of mathematicians. The Pythagoreans took vows of reciprocal assistance, shared their possessions, and pursued an ascetic lifestyle. They were often religious mystics, and occasionally wielded power as members of an aristocratic political faction in some cities of southern Italy. The school had two groups: the *mathematikoi* ("learners") who would theorize and develop new mathematical work, and the *akousmatikoi* ("listeners") who could only silently listen to Pythagoras' teachings behind a curtain. After Pythagoras' death, the two groups separated and developed different philosophical traditions.

Pythagoras used words and symbols to convey his thoughts. The *Tetractys*, shown in Figure 1, is a mystical symbol. It represents the decad $10 = 1+2+3+4$, the four classical elements (fire, air, water, earth), and the organization of space (from the zero-dimensional point to the three dimensional tetrahedron). It connects with the music of the spheres and the cosmos. As part of their initiation, the disciples took a secret oath mentioning the Tetractys as "nature's eternal fountain and supply".

The Pythagorean school also had a strong dogmatic slant. They are credited with the first known use of *ipse dixit* (autòs épha) as an argument from authority. The school was run as a sect, imposing great demands on the initiates. In *The Open Society and Its Enemies* (1945) the philosopher Karl R. Popper (1902–1994) warns

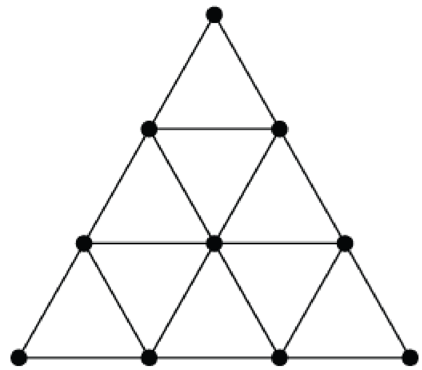

**Fig. 1** The Pythagorean Tetractys

about “tribalism, i.e. the emphasis on the supreme importance of the tribe without which the individual is nothing at all”, making a passing reference to the school in the comment that “the institution of tribal priest-kings or medicine-men or shamans [might] have influenced the old Pythagorean sect, with their surprisingly naive tribal taboos.”

Differently from the Pythagorean community, the mathematical tribe is not tribal at all. The reason is magisterially explained by Voltaire (1694–1778) in the entry “Sect” from his *Dictionnaire philosophique*: “There are no sects in geometry; nobody is spoken of as a Euclidean or an Archimedean. When the truth is apparent, it is impossible for parties or factions to arise.”

In fact, one could argue that the mathematicians are a rare example of a *global tribe*: they feel connected with each other across linguistic, political, geographical and even temporal barriers. (Perhaps only musicians share something comparable.)

A historical anecdote may be revealing. In 1919, after the First World War, the Allied Powers created the International Research Council (IRC) with the lofty mission to coordinate international scientific cooperation and to foster the formation of international scientific unions. The members of the Council, however, were not scientists or scientific associations but the governments of the Allied Powers, and the real objective was to curtail the primacy of German science.

On January 25, 1919 Magnus G. Mittag-Leffler (1846–1927) wrote a letter to Hardy declaring that “we as mathematicians need to be at the head in ‘the task of reestablishment of friendly relations’ between the men of science of all countries.” Nonetheless, IRC imposed that the mathematicians from the Central Powers (Germany, the Austro-Hungarian Empire, Bulgaria, and Turkey) be kept out of the quadrennial International Congress of Mathematicians held in 1920 and 1924.

The 1928 Congress was organized in Bologna. Salvatore Pincherle (1853–1936), president of the Congress and of the Italian Mathematical Union, maneuvered skillfully and gained access for mathematicians from all nations. In the August of 1928, David Hilbert (1862–1943) led a delegation of 76 German mathematicians to Bologna. It was the first time since the War that German scientists attended an international meeting. At the opening ceremony, they received a standing ovation and Hilbert firmly declared that “mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.”

## Recognition

In a short note, aptly titled “The mathematics tribe” [3], Dan Kalman shares a little game that he likes to play when attending a large scientific convention: he tries “to guess which of the passersby are mathematicians.” Claiming a pretty good average success, he writes that “there is a true feeling of community at the meetings: is it so strange that I can identify other members of my tribe?” Mathematicians know instinctively who their fellow tribespeople are.

Part of this connection originates in mathematics itself: a 2014 semi-anonymous post on the *xckd* forum reckons that “two mathematicians who have never met, who learned from entirely unrelated sources, may explain to one another precisely how they got to a conclusion and then agree upon its validity.” The same logical necessity that in Voltaire’s dictum dispels sectarianism binds mathematical minds *ex post*.

Sometimes the connection may mysteriously emerge even *ex ante*. More than a decade before that 2014 post, with strikingly similar words, Emmer [5] writes about the case of “two mathematicians who have never met [and], coming from different backgrounds and using different methods, reach the same result at the same time”. Ennio De Giorgi (1928–1996) and John Nash (1928–2015), born within few months of each other, independently solved Hilbert’s XIX problem between 1955 and 1956.

The significance of this coincidence is tersely captured in Nash’s own biographical information, prepared on the occasion of his being awarded the Nobel prize in Economics in 1994: “De Giorgi was first actually to achieve the ascent of the summit. [...] It seems conceivable that if either De Giorgi or Nash had failed in the attack on this problem [...] then that the lone climber reaching the peak would have been recognized with mathematics’ Fields medal”. (The Fields medal is arguably the highest honor conferred by the mathematical tribe; we return to it later.)

But there is more to mathematicians’ meeting of minds than mere logical necessity. The computer scientist Richard W. Hamming (1915–1998), winner of the 1968 Turing Award "for his work on numerical methods, automatic coding systems, and error-detecting and error-correcting codes" famously quipped that “the only generally agreed upon definition of mathematics is *Mathematics is what mathematicians do.*” The activity of doing mathematics connects mathematicians in a way that is difficult to describe or explain to non-mathematicians.

Where do such elusive links come from? Again, the photographer Cook shares her insight [1]: “I have photographed many people: artists, writers, and scientists, among others. In speaking about their work, mathematicians use the words ‘elegance,’ ‘truth,’ and ‘beauty’ more than everyone else combined.” Besides truth, mathematicians seek above all elegance and beauty. Many of them have written about beauty and mathematics; for example, Hardy argues that "beauty is the first test: there is no permanent place in the world for ugly mathematics."

Beauty and elegance are often impervious to outsiders. The mathematician Arthur Cayley (1821–1895) soberly reminds us that, "as for everything else, so for a mathematical theory: beauty can be perceived but not explained." Yet, mathematicians share similar mindsets about elegance and beauty in mathematics, producing widespread consensus on their occurrence. Perhaps surprisingly, this bold claim is supported by empirical evidence from the neurosciences.

An interdisciplinary team of four scientists, including Michael F. Atiyah (1929–2019, Fields medalist), used functional magnetic resonance imagining (fMRI) to classify the brain activity of 15 mathematicians engaged in contemplating mathematical formulae which they had individually rated for their beauty [6]. The results show that the experience of mathematical beauty correlates with activity in the same part of the brain as the experience of beauty derived from other sources. Among the 60 items used in the study, the popular Leonhard Euler's (1707–1783) identity

$$\mathrm{e}^{i\pi} + 1 = 0$$

was generally rated most beautiful, while Srinivasa Ramanujan's (1887–1920) infinite series

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}$$

was generally rated least beautiful.

Mathematicians appreciate elegance especially in theorems' proofs. Paul Erdős (1913–1996), arguably the most prolific author of mathematical papers ever, famously referred to *The Book* where God keeps the most elegant proof for each theorem—it takes a lot of hard work to be granted the honor of a short peek at The Book. Inspired by this evocative benchmark, in 1995 Aigner and Ziegler began to assemble some *Proofs from THE BOOK* with initial assistance from Erdős himself [7]. The result of their efforts, currently in its sixth edition, was awarded the 2018 Steele Prize for Mathematical Exposition because "this book does an invaluable service to mathematics, by illustrating for non-mathematicians what it is that mathematicians mean when they speak about beauty." This collection is a portfolio where curious minds who wish to peek what ticks mathematicians' sense of elegance can find plenty of examples.

## Ancestry

Mathematicians are keen about their ancestry. The youngest mathematician photographed by Cook is Maryam Mirzakhani (1977–2017, Fields medalist); see the paper by Strickland in this volume. During her interview, Mirzakhani "picked up a cup on her desk and began to talk about the shape of its handle, how that shape could be changed, and what mathematical questions and answers could be raised in the process." When Cook told her that another mathematician [Dennis Sullivan (1941–)] had explained topology exactly the same way, she exclaimed: "He's my mathematical grandfather!"

Mathematical kinship is not a blood relationship: it is a special link that ties the doctoral advisor (as a parent) with the advisee (as an offspring). Sullivan was the doctoral advisor of Curt McMullen (1958–, Fields medalist), who later became doctoral advisor for Mirzakhani: this makes her Sullivan's grand-niece. Learning to do mathematics often involves a long apprenticeship: mathematicians are very appreciative of the time and effort devoted by their professors, and in turn, they feel an obligation to nurture the next generation. Having a numerous progeny spanning generations is usually a badge of honor for a mathematician.

The matter is so important to mathematicians that they keep an official registry of record. The *Mathematics Genealogy Project* ( `genealogy.math.ndsu.nodak.edu`) provides online access to information about doctoral advisors and mathematical descendants for over 200,000 mathematicians. Because a genealogical tree is a directed graph, some mathematicians enjoy analyzing the mathematical structure of the database. In July 2016, for example, the genealogy graph had 200,037 vertices. There were 7639 (3.8%) isolated nodes and the largest component had 180,094 vertices (about 90% of all nodes). See [8] for more information.

The Genealogy Project describes a vertical relationship, usually relating older professors with younger students. Mathematicians are also fond of tracking collaborative relationships. Two or more mathematicians who publish a joint work are coauthors. The network of coauthorships is an undirected graph, where collaboration appears as a direct link; that is, $A$ and $B$ are connected if they are coauthors. Moreover, if a third mathematician has never coauthored with $A$ but is a coauthor of $B$, we say that their *collaborative distance* is 1. More generally, two mathematicians $A_0$ and $A_{n+1}$ have a collaborative distance $n$ if there are other $n$ distinct mathematicians $A_1,\dots,A_n$ who form a chain where $A_k$ and $A_{k+1}$ are coauthors, for $k = 0,1,\dots,n$.

This kind of horizontal network originated with reference to Erdős, who had about 500 coauthors over his vast production. When two people (who never worked together) wonder about the shortest collaborative path linking them, they are likely to discover that this path involves Erdős. If the collaborative distance between $A_i$ and Erdős is $d_i$, then the collaborative distance between $A_1$ and $A_2$ cannot be greater than $d_1 + d_2$. This led the mathematicians' tribe to brand each member $i$ with an *Erdős number* $d_i$, equal to the minimum collaborative distance between Erdős and $i$. After more than 20 years since Erdős' death, a large number of living mathematicians has a single-digit Erdős number. The online database for the *Mathematical Reviews*, run by the American Mathematical Society, has a tool to compute the collaborative distance between any two indexed authors, with a special option for the Erdős number.

Incidentally, movie buffs also tracks the collaborative distance between people who star in the same movie. The actor Kevin Bacon (1958–), who has starred in many movies spanning several genres, plays the same role as Erdős. The database `oracleofbacon.org` provides online access to the Bacon number for anyone who is indexed on IMDb, the *Internet Movie Database*. A mathematician who has appeared in a movie (or an actor who has coauthored a mathematical paper) is likely to have strictly positive Erdős and Bacon numbers. Aficionados enjoy exploring questions such as who has the lowest sum for these two numbers.

## 3 Language

To outsiders, mathematicians seem people who have a knack for borrowing ordinary words and return them with a meaning of their own making, often unrelated to common usage. Mathematicians' trees may have leaves and roots, but cannot cast any shadow and are not plants. And even their trees cannot grow square roots. A real number is no more real than an imaginary one. The Klein bottle cannot hold any liquid, and so on. Goethe wittily remarked: "Mathematicians are like Frenchmen: whatever you say to them they translate into their own language and forthwith it is something entirely different."

In fact, many insiders strongly believe that mathematics is a language in itself. Galileo Galilei (1564–1642) wrote that the universe "cannot be understood until we have learned the language and become familiar with the characters in which it is written. It is written in mathematical language." Josiah Willard Gibbs (1839–1903) was known as an unassuming scholar who rarely made public pronouncements; but, during a faculty meeting at Yale about replacing mathematics requirements for the bachelor's degree with foreign language courses, he rose and forcefully declared: "Gentlemen, mathematics is a language." R.L.E. Schwarzenberger (1936–1992) is adamant: "My own attitude [...] is simply that mathematics is a language. [...] It would be as foolish to attempt to write a love poem in the language of mathematics as to prove the Fundamental Theorem of Algebra using the English language" [9].

The power of the mathematical language lies in its precision and in its flexibility. In a conversation between the neuroscientist J.-P. Changeux and Alain Connes (1947–, Fields medalist), the former acknowledges that "mathematical language is plainly an authentic language" and immediately asks "but is it therefore the only authentic language?" Connes seeks precision, and magisterially corrects the question, while answering that "it is unquestionably the only universal language" [10].

Unfazed by the Babel myth, the tribe speaks one language across time and space for exchanging mathematical ideas among its members. Their system of communication combines natural language (such as English), technical jargon, symbolic notation and peculiar conventions. The natural language is only a substrate: taking it literally may generate misunderstandings, ranging from the serious to the hilarious; see [11] for a brilliant discussion.

The mathematical language evolves through the introduction of definitions, notation, and new terms (or different meanings for existing words). James J. Sylvester (1814–1897) was a mathematician with a passion for poetry and one of the most prolific contributors of mathematical neologisms, including *matrix* (introduced in 1850), *graph*, *invariant*, and *discriminant*. Charles L. Dodgson (1832–1898), a mathematician better known by his pen name Lewis Carroll, objected to Sylvester's choice for matrix in 1867: “I am aware that the word ‘Matrix’ is already in use [but] I use the word ‘Block’: [...] surely the former word means rather the mould, or form, into which algebraic quantities may be introduced, than an actual assemblage of such quantities.” Regardless of his literary merits, Carroll’s argument did not carry the day. But, as is for natural languages, mathematical words are driven by usage more than by merit: some expressions win, and others die out.

Some mathematical terms are associated with amusing anecdotes. Edward Kasner (1878–1955) was seeking a name for a very large number (1 followed by a hundred zeros, or $10^{100}$). During a walk with his two nephews, one of them suggested *googol*. The boy was sure that a googol was not an infinite number, and thus put forward *googolplex* for an even larger number. It was initially proposed that “a googolplex should be 1, followed by writing zeros until you get tired.” But eventually the matter was settled by defining a googolplex to be $10^{\text{googol}}$, or $10^{10^{100}}$.

The term *random variable* competed for some time against *chance variable* and *stochastic variable*. Apparently, the issue was solved by Chance itself. Joseph L. Doob (1910–2004) recalls that he had an argument with William Feller (1906–1970) at the time when they were writing their monographs: “He asserted that everyone said ‘random variable’ and I asserted that everyone said ‘chance variable’. We obviously had to use the same name in our books, so we decided the issue by a stochastic procedure. That is, we tossed for it and he won.” [13].

Notation is another important piece of the mathematical language. It can compress information, and thus makes it easier to advance thinking by building on existing knowledge. Pierre-Simon de Laplace (1749–1827) argues that “such is the advantage of a well constructed language that its simplified notation often becomes the source of profound theories.” Alfred N. Whitehead (1861–1947) adds that “by relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, increases the mental power of the race.”

The Pythagorean Tetractys in Figure 1 compresses information in an arcane way. Good notation, instead, feels natural to the mind: mathematicians speak about the elegance and beauty of their notation. But it may be difficult to get things right. For a long time equations were proposed and solved in words, making it difficult to generalize the results. François Viète (1540–1603) initiated a conscious effort to create powerful notation and began to write simpler structured expressions such as

2 in A quad – 3 in A plano + 4 aequatur 0

In less than a century, with important contributions from René Descartes (1596–1650), the mathematicians learned to write

$$2x^2 - 3x + 4 = 0$$

Most outsiders find it hard to believe that most of the mathematical notation we take for granted nowadays did not exist only a few centuries ago. For instance, the ubiquitous $\pi$ (i.e., the ratio of the circumference of a circle to its diameter) is unostentatiously introduced by William Jones (1675–1749) only in 1706. Until the XV century, the common notation for addition and subtraction in Europe uses P (plus) and M (minus). The modern symbols $+$ and $-$ first appear in print in 1489, with reference to surplus and deficit in business problems. The symbol $\surd$ for square root shows up in 1525; Descartes adds the vinculum in 1637, changing it to $\sqrt{\phantom{x}}$. The inequality signs ($>$ and $<$) are from the XVII century. Union and intersection ($\cap$ and $\cup$) appear in the XIX century, and $\emptyset$ is introduced in the XX century.

Because of the variety of symbols, mathematical typesetting used to be considered laborious and highly error-prone. This changed drastically after Donald Knuth (1938–) released TeX in 1978. This typesetting language was designed to generate exactly the same results under any operating system, by keeping distinct the source code prepared by the author and the generated output. The source code describes what the author wants to achieve, leaving the gritty work of delivering it to the computer. Knuth put TeX in the public domain, allowing many other people to expand it and make it into the most sophisticated digital typesetting system currently available.

TeX and its later variants (most notably, LaTeX) grant the user full control on the appearance of a document, so that one can produce high-quality output with relatively low effort. TeX-based typesetting is especially popular among mathematicians and many other scientific communities, who strive for accuracy or use technical notation. For a modest example, this article has been typeset by the author in LaTeX, using macros designed by the publisher for ensuring consistent results across the whole book. (Incidentally, this makes both the author and the publisher happier: the former has a reasonable amount of control on the final output, and the latter makes huge savings on the typesetting costs.)

One important byproduct of the popularity of TeX is that most mathematicians may write and read the source code: they can use it to communicate their notation in writing—over email or other systems—with the same accuracy that they devote to their definitions and their theorems. The sender may compose the text

```
Dear Colleague, I have just proved that
  x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}
```

and the receiver would read

Dear Colleague, I have just proved that

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

A showcase for the amazing typesetting abilities of TeX is at `www.tug.org/texshowcase`. We are especially fond of `diminuendo.tex`, that generates decimal expansions for a few prominent rational, irrational and transcendental numbers, making the digits progressively smaller so that the decimal expansion fits a finite area.

Budding mathematicians must learn the language of mathematics to become effective members of the tribe. As for natural languages, this is more frequently done through direct interactions with other mathematicians than by poring over books. These interactions foster a culture and a sense of community to which we return in Section 4.

An ingenious feature of the mathematical language is that its terms may have different denotations in different contexts, making it possible to unify different phenomena under one roof. The most obvious example is the conflation of numbers (quantities) and geometric figures: for instance, the Pythagorean theorem states at the same time a relationship between three geometric squares and an equation involving three numbers. The same sentence carries both a geometric and an arithmetic interpretation, respectively shown on the left and on the right in Figure 2.

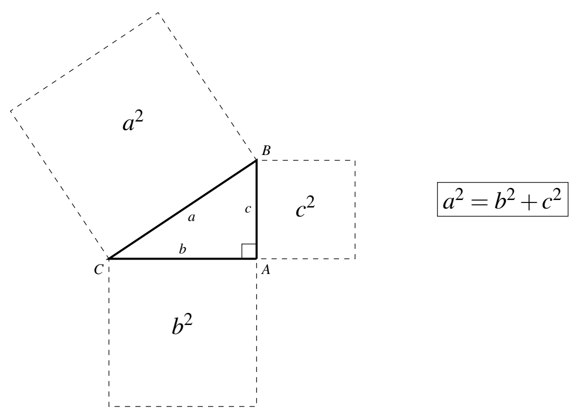


**Fig. 2** The Pythagorean theorem.

Henri Poincaré (1854–1912) explained that "Mathematicians do not study objects, but relations between objects. Thus, they are free to replace some objects by others so long as the relations remain unchanged. Content to them is irrelevant: they are interested in form only." Focusing on relations provides a shortcut to enrich the language with analogies. For example, compare the distributive property for addition and multiplication

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

and the distributive property for union and intersection

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

We may only see $\cap$ replacing $\cdot$ and $\cup$ replacing $+$, but this superficial difference hides a common structure that is apparent to mathematicians. As Poincaré quipped: "Mathematics is the art of giving the same name to different things."

Language alone is not enough for doing mathematics. Richard Feynman (1918–1988) warns us that "Mathematics is not just a language. Mathematics is a language plus reasoning. It's like a language plus logic. Mathematics is a tool for reasoning. It's, in fact, a big collection of the results of some person's careful thought and reasoning."

Mathematical language without an underlying reasoning turns into a parody. In the movie *The Wizard of Oz*, the Scarecrow asks for a brain and, when he finally gets one, he claims that "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side." This is nonsense wrapped into mathematical language. The scene is paraphrased in the opening scene of Episode 10, Season 5 of the animated sitcome *The Simpsons*, with Homer Simpsons uttering the same sentence.

## 4 Culture

The mathematical community shares a culture that includes: (a) social organization; (b) traditions, rituals, and folklore; (c) accomplishments that the tribe esteems so highly to give their doers the status of heroes. We offer only vignettes for each of them, because lack of space prevents us from giving a detailed description.

### Social organization

The mathematical tribe is open-minded and cooperative. Social ties are loose, but existent. Our photographer-guide Cook has two insights to contribute. The first is that "Truth is the ultimate authority in mathematics." Most claims can be settled as either true or false, and this reduces the potential for in-fighting or doctrinal clashes.

The second insight is that "Mathematicians are bound by fairness. Anyone who solves an outstanding problem with a pencil and piece of paper [...] can be catapulted into the upper echelons of the mathematical community overnight. [...] Mathematics may well be the most democratic of creative pursuits, as is the recognition of success by fellow mathematicians. Honesty and conscience are the tools of character required." The unwritten rule is that most mathematicians quarrel like all human beings, but they are expected to concede an argument when it becomes clear what truth demands.

Beyond this common attitude, there are of course different styles. In the language of anthropology, one might speak of *bands* belonging to the same tribe. David Mumford (1937–, Fields medalist) gave an interesting categorization in a blog piece titled "Math & Beauty & Brain Areas", dated 11 October 2015. He classifies mathematicians into four bands, according to what most strongly drives them.

The *explorers* enjoy "discovering what lies in some distant mathematical continent and, by dint of pure thought,' shining a light and reporting back. Some of them are *gem collectors* who undig wholly new objects; others are *mappers* who describe the new continents.

The *alchemists* enjoy "finding connections between two areas of math that no one had previously seen as having anything to do with each other."

The *wrestlers* focus "on relative sizes and strengths of this or that object. They thrive [...] on inequalities, and on asymptotic estimates of size or rate of growth."

Finally, the *detectives* "doggedly pursue the most difficult, deep questions, seeking clues here and there, [...] often searching for years or decades." Some of them are *strip miners* who "are convinced that underneath the visible superficial layer, there is a whole hidden layer and that the superficial layer must be stripped off to solve the problem." Others are *baptizers* "who name something new, making explicit a key object [...] whose significance is clearly seen only when it is formally defined and given a name."

Besides Mumford's loose characters, the mathematical tribe also has formal structures in place. The most important one is probably the *2010 Mathematics Subject Classification* (MSC2010) "produced jointly by the editorial staffs of Mathematical Reviews (MR) and Zentralblatt für Mathematik (Zbl) in consultation with the mathematical community". This classification index is used to tag items in the mathematical literature and help "users find the items of present or potential interest to them as readily as possible". The MSC2010 is a hierarchical scheme, with three levels of structure: at the top level only, it already recognizes 64 distinct mathematical disciplines.

Moving to political bodies, the mathematicians' equivalent of the United Nations is the *International Mathematical Union* (IMU). This is an international non-governmental organization that promotes international cooperation in the field of mathematics. Its members are the national mathematics organizations from more than 80 countries.

IMU supports the International Congress of Mathematicians (ICM) and acknowledges outstanding mathematical research through the awarding of scientific prizes. Its history has been affected by the political controversies after World War I mentioned in Section 2: the IMU was established in 1920, dissolved in September 1932, and finally reinstated in 1951 with the initial membership of ten countries. Since 2011, its permanent offices are located in Berlin.

In 2006, the International Mathematical Union (IMU) announced its adoption of a new logo. The logo design is visible at `www.mathunion.org/outreach/imu-logo`: it is based on the Borromean rings, a famous topological link of three components with the property that, if any one component is removed, the other two fall apart (while all three together remain linked). The designer says that this logo "represents the interconnectedness not only of the various fields of mathematics, but also of the mathematical community around the world."

The *International Congress of Mathematicians* (ICM) predates the IMU. Held every four years, this is the most significant meeting in pure and applied mathematics. It is also one of the oldest scientific congresses: the first ICM took place in Zurich in 1897, at a time where several scientists began an effort to make science transcend political boundaries. Mathematicians, who have a keen sense of being a community, were at the forefront of this effort.

## Traditions, rituals and folklore

The standard instruction offered in many schools is different from the initiation rituals for budding mathematicians. One enters the mathematical community by cooptation: if “mathematics is what mathematicians do”, then a mathematician is likely to be someone who is accepted as such by other mathematicians. A lot of mathematical knowledge is passed during a period of apprenticeship, where the candidate learns by direct experience and oral transmission which implicit knowledge and hidden assumptions populate the conversations of active mathematicians in the field of the candidate. The creation of mathematics may need to access unbridled ideas; meticulous proofs are often written only after a result has already been uncovered. In passing, we note that mathematical knowledge is often ahistorical: Hilbert reportedly quipped that “one can measure the importance of a scientific work by the number of earlier publications rendered superfluous by it” [15].

Contrary to popular belief, mathematics is often a collaborative effort. Mathematical ideas are nurtured by bouncing them between different minds. Whenever possible, mathematicians congregate to facilitate this process. Many countries have international research centers where mathematical scholars from all over the world meet over more than a few days: they pursue research by discussing recent developments with their colleagues, and in so doing often generate new ideas or open new perspectives.

Modern technology, of course, allows effective means of telecommunication, from email to video calls. In the old times, many mathematicians wrote letters. Some of them gave us a window on how the bonds tying the tribe’s members were able to overcome time and distance. At a time where many scientists had a sex prejudice, Marie-Sophie Germain (1776–1831) had a correspondence with famous mathematicians such as Joseph-Louis Lagrange (1736–1813), Adrien-Marie Legendre (1752–1833), and Gauss (1777–1855).

Fearing rejection because of her sex, she approached Gauss under the pen name of Monsieur LeBlanc. When Gauss discovered who she really was, he had no hesitation in acknowledging her merits: “when a woman, because of her sex, our customs and prejudices, encounters infinitely more obstacles than men, in familiarizing herself with their knotty problems, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent, and superior genius.” One year younger than Germain, but far more influential, Gauss pressured the University of Göttingen to grant her a (posthumous) honorary degree in 1837 and is now listed in the Mathematics Genealogy Project as her “father”.

The correspondence between Pierre de Fermat (1607–1665) and Blaise Pascal (1623–1662) is often celebrated as the founding moment for the modern theory of probability [14]. The first letter was sent out by Pascal on August 24, 1654; it is a testimony to how much the best mathematicians feel that truth is the ultimate authority: “I wish to lay my whole reasoning before you, and to have you do me the favor to set me straight if I am in error or to endorse me if I am correct.”

Coffee is a beverage that mathematicians often associate with collaborative efforts and intense research. The Lviv School of Mathematics, founded by Hugo Steinhaus (1887–1972) and Stefan Banach (1892–1945), enlisted many capable mathematicians who used to meet and work at the Scottish Café. The problems discussed at this (Polish) coffee shop were collected in a thick notebook provided by Banach's wife, that came to be known as the *Scottish Book*. Stanislaw Ulam (1090–1984) was one of the major contributors. In 1957, he received from Steinhaus a copy of the notebook which had survived the war, and translated it into English [16]. Later on, Erdős, an avid coffee-drinker who used it to sustain prolonged efforts at mathematics, gave this beverage a special status in the tribe's collective memory with his memorable definition: "A mathematician is a machine for turning coffee into theorems."

A disappearing ritual that many mathematicians are still fond of practicing, and sometimes strenuously defend, may be called the *chalkboard dance*. Some mathematicians passionately argue that teaching requires using the whole body, gesturing and pausing, up to becoming one with the board. (This is contrasted with the dull activity of pushing a button to change slides.) Israel M. Gelfand (1913–2009) conjectures that gesturing may contribute to making frontal teaching more natural: "using chalk on a blackboard, we write by moving the entire arm. [...] Wider movements of arms fit naturally in the cycle of breathing and speaking."

In front of a blackboard—more than one is even better—a mathematician can trace and link ideas as quick as they appear to the mind. There seems to be a connection between the agility of a mind and the time necessary to make them visible (and communicate them). It is not by chance, perhaps, that mathematics makes recourse to motion and action metaphors: asymptotes approach, limits converge, variables run, and so on. The chalkboard dance is not popular among teaching innovators and university administrators, but—when sipping their coffee—several mathematicians speak lovingly of it.

Renteln and Dundes [17] bring a joking viewpoint about the traditions of the mathematical tribe by observing that a folk is a group that shares at least one common factor. "Hence, mathematicians constitute a folk. And, like all folk groups, [they] have their own folk speech (slang), proverbs, limericks, and jokes, among other forms of folklore." Some of this folklore is esoteric, because outsiders may not have the requisite knowledge to appreciate it. Their article offers a generous sampling of "both esoteric and exoteric mathematical folklore, concentrating on humorous genres such as jokes."

Another set of traditions brings mathematics closer to a sport, in the sense of an activity involving exertion and skill, in which an individual competes against another or others. Some competitions are lonely races, where one competes against a very difficult problem or a long-standing conjecture: for example, Andrew Wiles (1953–) worked over six years in secrecy to prove Fermat's last theorem. He eventually reached his goal at the age of 41, too late to being awarded the Fields medal (restricted to those under 40): but the achievement was so momentous that the International Mathematical Union (1998) recognized it with a silver plaque in 1998.

More traditional competitions take the form of challenges: in the past, these could be public matches like the one opposing Niccolò Fontana Tartaglia (1499/1500–1557) to Ludovico Ferrari (1522–1565) over the solution of cubic equations. Other disputes, often ferocious and sometimes prolonged, take place when people advance conflicting priority claims over mathematical issues. The most famous involved Isaac Newton (1643–1727) and Gottfried Wilhelm Leibniz (1646–1716) over the development of infinitesimal calculus.

Nowadays, challenges are often issued as open problems. The most influential (implicit) example is Hilbert's list of ten problems, originally presented in 1990 at the International Congress of Mathematicians in Paris, and later expanded to a complete list of 23 problems published in 1902. (Some of them are still open.)

A recent popular example are the seven Millennium Prize Problems published in 2000: a correct solution to any of them carries a one million U.S. dollars prize funded by the Clay Mathematics Institute. Until now, only one has been solved: the Poincaré conjecture has been established by Grigori Perelman (1966–), who has successively declined both the prize money and the Fields medal. It is worth reading about his personality and the reasons for this unusual decision; see [18].

A very different kind of competition is the International Mathematical Olympiad (IMO), a world mathematics contest for high school students selected through local and national competitions. It is the oldest of the International Science Olympiads. The first IMO was held in 1959 in Romania, with 7 countries participating. Since then, the competition has taken place every year (except 1980) and has grown to include over 100 countries from five continents. Every country can send up to six contestants, who must be under the age of 20 and cannot be registered at any tertiary institution. The logo of the International Mathematical Olympiad is arguably more elegant than the icon of the Olympic games: it weaves zero and infinity, using the same five colors to denote the five continents but making sure that each color is in touch with all the other four (in the Olympic icon, each color touches at most other two). The IMO logo can be seen at `www.imo-official.org`.

The International Mathematical Olympiad is a showcase for young talents, encouraging them to pursue mathematics. The selection process is used by many countries to screen and nurture the young talents, who often make rapid and significant advances in the ranks of the tribe. For instance, to date fifteen IMO participants have successively been awarded the Fields medal. (Perelman was also a IMO contestant, but he declined the award.)

The participants are ranked individually based on their scores. Medals are awarded to the highest ranked participants but, differently from the Olympic Games, about half of them is awarded a medal. The numbers of gold, silver and bronze medals are approximately in the ratios 1 : 2 : 3; that is, the first twelfth of the participants receives a gold medal, the next sixth is given a silver medal, and the next fourth is awarded bronze medals. Anyone who scores above a threshold on at least one problem receives an honorable mention. The tribe does not wish to single out one young person above another, because the advancement of mathematics is a collective enterprise.

Mathematicians are probably the scientific group who confer more awards than any other one. However, the real awards never follow an explicit competition that inevitably declares at the same time a winner and at least one loser. The mathematical awards are offered to recognize brilliant minds whose achievements shine light for all. They are meant to celebrate the best, but the coreography tries to encourage everyone to join in. (Notwithstanding this, it is clear that some non-winners experience disappointment.)

Out of the many existing mathematical prizes, we mention only two. Named after the Norwegian mathematician Niels Henrik Abel (1802–1829), the *Abel Prize* is awarded annually by the King of Norway to one or more outstanding mathematicians. It is modeled after the Nobel Prizes and comes with a monetary award of 6 million Norwegian Kroner. Its history dates back to 1899 when, upon learning that Alfred Nobel's plans would not include a prize in mathematics, Sophus Lie (1842–1899) proposed to establish a dedicated award. In 1902 King Oscar II of Sweden and Norway seemed open to finance a mathematics award to complement the Nobel Prizes, but the matter was sidetracked by the dissolution of the union between Norway and Sweden in 1905. The Abel Prize was finally established in 2001 "to give the mathematicians their own equivalent of a Nobel Prize."

The Abel Prize is awarded by a committee appointed by the Norwegian Academy of Science and Letters. The tribe is not directly involved in the choice. The Fields medal, instead, is conferred at the International Congress of Mathematicians, in front of the convened mathematicians. This circumstance and its longer history—the award was first conferred in 1936—may be the reasons why the Fields medal is generally reputed more prestigious, in spite of carrying a monetary prize of only 15,000 canadian dollars. Since 1966, the Fields medal is conferred every four years to no more than four mathematicians under the age of 40. The youngest winner was 27 years old when he received the award. The obverse of the medal depicts Archimedes (3rd century BC); the reverse has an inscription in Latin that translates to "Mathematicians gathered from the entire world have awarded [this prize] for outstanding writings." The community honors its champions.

## Heroes

The champions are noted or admired for their outstanding achievements. Sometimes, the tribe elevates a champion to the rank of hero, who transcend ordinary mathematicians in creativity, technical skill, or vision. The feats of a hero fuel narratives that are part of the mathematical culture, similar to the role played by the founding myths in other cultures.

There is a rich pantheon of mathematical heroes, and opinions on who are the most important ones are unlikely to be unanimous. We offer a few representative samples, arranged by birthdate. Three great heroes from the classical age are Pythagoras, Euclid (4th century BC), and Archimedes. Four giants from the modern age are Newton, Leibniz, Euler, and Gauss. One might of course conceive many other names, especially in connection to specific fields. For instance, a triad of heroes for probability theory are Fermat, Pascal and Jacob Bernoulli (1655–1705).

Naming contemporary heroes is fraught with causing controversies, so we deflect this issue and mention three heroes whose stories became successful movies making their names known to the general public, outside of the tribe. The feature film *The Man Who Knew Infinity* (2015) portrays the life of Srinivasa Ramanujan (1887-1920), based on the eponymous biography published in 1991. The drama film *The Imitation Game* (2014) introduces Alan Turing (1912–1954), after the biography *Alan Turing: The Enigma* (1983); see [19]. And the biographical drama film *A Beautiful Mind* (2001), adapted from the eponymous biography published in 1998, portrays the life of John Nash; see [20]. He missed the Fields medal but traveled to Scandinavia in at least two occasions, collecting both the Nobel prize in Economics (1994) and the Abel prize (2015). (Besides, the movie won four Academy Awards, popularly known as Oscars, including one for Best Picture.)

One could find many ways to add to our short list. For instance, the protagonist of the political satire film *Dr. Strangelove or: How I Learned to Stop Worrying and Love the Bomb* (1964), loosely inspired by the thriller *Red Alert*, is partly fashioned after by John von Neumann (1903–1957). We choose two cases.

The first choice is meant to honor the contribution of women mathematicians. The biographical drama film *Hidden Figures* (2016), loosely based on the non-fiction eponymous book, dramatizes the story of the black female mathematicians who worked at the National Aeronautics and Space Administration (NASA) during the Space Race. Twice discriminated (by gender and by color), these women where hidden figures in a white male world; see the paper by Emmer in this volume.

The second choice is meant to honor the scholars who over many centuries have opened up whole new territories for the mathematical tribe. For at least two millennia, the philosophers have agonized over the difference between potential vs actual infinity, often negating (for theological reasons) the existence of the latter. Georg Cantor (1845-1918) discovered that it is possible to compare the size (in technical jargon: the cardinality) of infinite sets, establishing a hierarchy of infinite numbers. If Ramanujan knew infinity, Cantor faced it and found a way to count beyond infinity!

It takes an amazing courage to look across the infinity, where our (finite) intuition no longer supports the mind. When Cantor proved that a segment had the same number of points as a square, he candidly wrote in a letter to J.W. Richard Dedekind (1831–1916): “I can see it, but I don’t believe it!” (June 20, 1877). His theory was initially met by opposition from within the tribe, including Poincaré who firmly declared: “There is no actual infinity. The Cantorians forgot this, and so have fallen into contradiction.” But in the end truth reaffirmed its authority and his theory was accepted. Another hero, David Hilbert, acknowledged Cantor’s unique achievement in 1926 with a lapidary statement: “No one shall expel us from the paradise that Cantor has created.”

**Acknowledgment.** Without implications, I owe Gabriele Lolli a debt of gratitude for our conversations about the tribe. I thank Giulia and the editors for their comments.

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