Market allocations under conflation of goods

Online Appendix

(available at `http://mizar.unive.it/licalzi/MACG-Online-Appendix.pdf`)

1 Pareto-dominated configurations

We show one way to formulate Proposition 4 in measure-theoretic terms. Intuitively, this requires that the endowment of goods is well-distributed, in the sense that the measure ω is non-atomic. Let $\Pi_{(=k)}$ be the set of the classifications using *exactly* k. Given π in $\Pi_{(=k)}$, rearrange it in the string of intervals $(F_1, \ldots, F_k)$ where $i < j$ if and only if $s < t$ for every s in F_i and t in F_j. If we associate π with the vector $\omega(\pi) = (\omega(F_1), \ldots, \omega(F_k))$ in $\mathbb{R}^k$, then the map $\pi \mapsto \omega(\pi)$ is continuous and injective. Define the measure $\lambda(O)$ for a set $O \subset \Pi_{(=k)}$ as the Lebesgue measure of the set $\{\omega(\pi) : \pi \in O\}$.

Corollary 8. *Suppose that agents are not all identical and $k > 1$. Then there is a non-null subset O of classifications in $\Pi_{(=k)}$ whose competitive configurations are Pareto-dominated by some competitive configuration $\langle \hat{\pi}, (\hat{x}^i) \rangle$ with $\hat{\pi}$ in $\Pi_{(=k)}$.*

The following example illustrates a society with two agents, where every configuration can be improved both from a Paretian and an utilitarian point of view with a suitable refinement of the underlying classification.

Example 7. *Consider an economy where ω is the Lebesgue measure and there are two agents with identical claims. Let $S \subset \mathcal{I}$ denote the Smith-Volterra-Cantor set (SVC set for short), which is a measurable set of size $\frac{1}{2}$ with the property that every non-null interval in $\mathcal{I}$ contains a non-null interval disjoint from S; see the ϵ-Cantor set in Aliprantis and Burkinshaw (1981, p. 141). Agents' of each group have linear preferences based on the evaluation measures:*

$$\nu_1(F) = 2\omega(F \cap S), \quad \nu_2(F) = 2\omega(F \setminus S).$$

We claim that the only Pareto-optimal configurations assign the 0 *bundle to all agents of group* 1.

Let $\langle \pi, (x^i) \rangle$ be a configuration and let the interval $B \in \pi$ be a commodity such that $x^1_B > 0$. By the properties of the SVC set S, there exists an interval $C \subseteq B$ such that

$C \cap S = \emptyset$, and so $\nu_1(C) = 0$ and $\nu_2(C) = 2\omega(C)$. If we label C as a new commodity, we obtain a finer classification ρ under which one can transfer all goods of type C previously assigned to 1 to agent 2, while leaving the rest of the allocation unchanged. But this benefits agent 2 without causing harm to 1 (because her evaluation of C is null), proving that $\langle \pi, (x^i) \rangle$ is Pareto-dominated.

Formally, let B^1 and B^2 be the two (possibly, empty) intervals obtained by removing C from B. Let ρ be a refinement of the classification π, where the commodity B has been replaced with B^1, B^2 and C. Consider a new allocation (y^i) in $\mathcal{E}(\rho)$ where the bundle assigned to agent 1 is

$$y_A^1 = \frac{\omega(A)}{\omega(B)} x_B^1 \text{ if } A \in \{B^1, B^2\}, \quad y_C^1 = 0, \quad y_A^1 = x_A^1 \text{ otherwise;}$$

and the bundle assigned to agent 2 is:

$$y_A^2 = \frac{\omega(A)}{\omega(B)} x_B^2 \text{ if } A \in \{B^1, B^2\}, \quad y_C^2 = \frac{\omega(C)}{\omega(B)} x_B^2 + \frac{\omega(C)}{\omega(B)} x_B^1, \quad y_A^2 = x_A^2 \text{ otherwise.}$$

Computations shows that (y^i) is a feasible allocation in $\mathcal{E}(\rho)$ that agent 1 finds equivalent to (x^i), while agent 2 strictly prefers it to (x^i). Standard arguments based on the continuity and monotonicity of the function $V_i(\rho, \cdot)$ prove that one can modify (y^i) into a new allocation that every agent strictly prefers to (x^i).

Clearly, Example 7 relies crucially on the assumption that agents' evaluations of goods are expressed through extremely elaborated subsets of $\mathcal{I}$ (such as the SVC set) while commodities can only be defined as intervals. If we allow commodities to be arbitrary subsets of $\mathcal{I}$, then the classification $\pi = \{S, S^c\}$ would generate a Pareto-optimal configuration where all goods of type S are assigned to agent 1 and the rest to agent 2. This suggests that the stronger the exogenous constraints on the classification of goods into commodities, the further agents may be from reaching optimal allocations.

2 Comparative statics with opposed preferences

Given the classification $\pi = (C_1, \ldots, C_k)$ where intervals are naturally ordered, let $p = (p(C_i))_i$ denote the system of competitive equilibrium prices, with $x = (x(C_i))_i$ and $y = (y(C_i))_i$ being the equilibrium bundles respectively assigned to 1 and 2. We outline the main steps, including the explicit computation of $\frac{\partial V_1^*(\pi)}{\partial \theta_1}$.

1. The ratio $\nu_1(C_i)/\omega(C_i)$ decreases as i increases. Indeed, let $F(x) = \int_0^x f(t)\,dt$, so that $F'(x) = f(x)$. Assume $a < b < c$; given $A = (a, b)$ and $B = [b, c)$, by the Mean Value theorem $\nu_1(A)/\omega(A) = (F(b) - F(a))/(b - a) = f(\theta_A)$ for some θ_A in A, and similarly $\nu_1(B)/\omega(B) = (F(c) - F(b))/(c - b) = f(\theta_B)$ for some θ_B in B. When f is decreasing, $f(\theta_A) > f(\theta_B)$ and thus $\nu_1(A)/\omega(A) > \nu_1(B)/\omega(B)$. Specularly, $\nu_2(C_i)/\omega(C_i)$ increases as i increases.

2. Suppose that x is an equilibrium allocation. If

$$\frac{p(C_j)}{p(C_i)} > \frac{\nu_1(C_j)/\omega(C_j)}{\nu_1(C_i)/\omega(C_i)},$$

then $x(C_j) = 0$. Correspondingly, $x(C_i) > 0$ and $x(C_j) > 0$ requires that the above expression holds as an equality. Similar considerations hold for y and ν_2.

3. There is a j^* such that $x(C_i) = \omega(C_i)$ for $i < j^*$ and $y(C_j) = \omega(C_j)$ for $j > j^*$.

Proof. Suppose $i < j$ and $x(C_j) > 0$. Then (2) gives

$$\frac{p(C_j)}{p(C_i)} \leq \frac{\nu_1(C_j)/\omega(C_j)}{\nu_1(C_i)/\omega(C_i)} < 1,$$

where the last inequality follows from (1). Using (1) again gives

$$\frac{p(C_j)}{p(C_i)} < 1 < \frac{\nu_2(C_j)/\omega(C_j)}{\nu_2(C_i)/\omega(C_i)}$$

and thus $y(C_i) = 0$, or (2) would be violated. Therefore, because the market clears in equilibrium, $x(C_i) = \omega(C_i)$. $\square$

4. Let $C^* = C_{j^*}$ be the disputed commodity; define $C_\ell = \bigcup_{i<j^*} C_i$ and $C_r = \bigcup_{j>j^*} C_j$. If $x(C^*) > 0$ and $y(C^*) > 0$, then $x(C^*) = \xi\omega(C^*)$, where

$$\xi = \frac{1}{2}\left[\frac{\nu_2(C_r)}{\nu_2(C^*)} - \frac{\nu_1(C_\ell)}{\nu_1(C^*)} + 1\right]. \qquad (*)$$

Proof. By (1), we have $x(C_i) = \omega(C_i)$ for $i < j^*$ and $x(C_j) = 0$ for $j > j^*$. Applying (2) gives

$$\frac{p(C^*)}{p(C_i)} = \frac{\nu_1(C^*)/\omega(C^*)}{\nu_1(C_i)/\omega(C_i)} \quad \text{for every } i < j^*,$$

from which we obtain

$$\omega(C_i)p(C_i) = \left[\frac{\omega(C^*)p(C^*)}{\nu_1(C^*)}\right]\nu_1(C_i) \quad \text{for every } i < j^*.$$

Given the system of prices p, the worth of the bundle x for Agent 1 is:

$$\sum_{i<j^*} \omega(C_i)p(C_i) + \xi\omega(C^*)p(C^*) = \left[\frac{\omega(C^*)p(C^*)}{\nu_1(C^*)}\right] \cdot \left[\nu_1(C_\ell) + \xi\nu_1(C^*)\right].$$

Similarly, the worth of the bundle y for Agent 2 is:

$$\sum_{j>j^*} \omega(C_j)p(C_j) + (1-\xi)\omega(C^*)p(C^*) = \left[\frac{\omega(C^*)p(C^*)}{\nu_2(C^*)}\right] \cdot \left[\nu_2(C_r) + (1-\xi)\nu_2(C^*)\right],$$

Because in equilibrium x and y must have the same worth at p, we have:

$$\frac{\nu_1(C_\ell) + \xi\nu_1(C^*)}{\nu_1(C^*)} = \frac{\nu_2(C_r) + (1-\xi)\nu_2(C^*)}{\nu_2(C^*)}$$

from which (*) follows. □

5. By a standard continuity argument:

$$x_{j^*} = \begin{cases} 0 & \text{if } \xi \le 0, \\ \xi\omega(C_{j^*}) & \text{if } 0 < \xi < 1, \\ \omega(C_{j^*}) & \text{if } \xi \ge 1, \end{cases}$$

with $y_{j^*} = \omega(C_{j^*}) - x_{j^*}$.

6. Suppose $C^* = (\theta_1, \theta_2)$ and denote by V_1 the utility that agent 1 obtains from the bundle x. Using (5), we have

$$V_1^* = \begin{cases} \nu_1(C_\ell) & \text{if } \xi \leq 0, \\ \nu_1(C_\ell) + \xi \nu_1(C^*) & \text{if } 0 < \xi < 1, \\ \nu_1(C_\ell \cup C^*) & \text{if } \xi \geq 1. \end{cases}$$

In particular, when $0 < \xi < 1$, substituting (*) from (4) gives:

$$V_1^* = \frac{\nu_1(C_\ell) + \nu_1(C^*)}{2} + \frac{\nu_1(C^*)\nu_2(C_r)}{2\nu_2(C^*)} = \frac{\nu_1(C_\ell \cup C^*)}{2} + \frac{\nu_1(C^*)\nu_2(C_r)}{2\nu_2(C^*)}.$$

Recall that $C_\ell = [0, \theta_1]$, $C^* = (\theta_1, \theta_2)$ and $C_r = [\theta_2, 1]$. Therefore:

$$\frac{\partial \nu_1(C_\ell \cup C^*)}{\partial \theta_1} = 0, \qquad \frac{\partial \nu_1(C^*)}{\partial \theta_1} = -f_1(\theta_1), \qquad \frac{\partial \nu_2(C^*)}{\partial \theta_1} = -f_2(\theta_1).$$

Then the derivative of V_1^* with respect to θ_1 when $0 < \xi < 1$ is:

$$\frac{\partial V_1^*}{\partial \theta_1} = \frac{\nu_2(C_r)}{2\nu_2^2(C_{j^*})} \left[\nu_1(C_{j^*}) f_2(\theta_1) - f_1(\theta_1)\nu_2(C_{j^*}) \right].$$

3 Refinements may not be welfare-improving

The next example exhibits an economy and a classification π with the following property: for every (finer) classification ρ that splits a commodity from π into two commodities, there is an agent who strictly prefers every competitive allocation in $\mathcal{E}(\pi)$ to any competitive allocation in $\mathcal{E}(\rho)$. In short, adding a new commodity damages at least one agent and therefore is not a Pareto-improvement for the society.

Example 8. *Consider an economy where ω coincides with the Lebesgue measure. There are 4 agents with identical claims and linear preferences based on the evaluation measures:*

$$\nu_1(F) = 2\omega\left(F \setminus \left[\frac{1}{4}, \frac{3}{4}\right]\right), \quad \nu_2(F) = 2\omega\left(F \cap \left[\frac{1}{4}, \frac{3}{4}\right]\right),$$
$$\nu_3(F) = 2\omega\left(\left[0, \frac{1}{2}\right]\right), \quad \nu_4(F) = 2\omega\left(\left[\frac{1}{2}, 1\right]\right).$$

Let π be the classification formed by the two intervals $A = \left[0, \frac{1}{2}\right]$ and $B = \left(\frac{1}{2}, 1\right]$.

In the exchange economy $\mathcal{E}(\pi)$ agent 3 cares only about commodity A, agent 4 only about B, and agents 1 and 2 are indifferent between them. An equilibrium is achieved when the two commodities have the same price and agents demand, for example, the π-bundles:

$$x^1 = x^3 = \left(\frac{1}{4}, 0\right), \; x^2 = x^4 = \left(0, \frac{1}{4}\right).$$

We claim that for every refinement ρ of π formed by 3 tradable commodities there is an agent that strictly prefers (x^i) to any competitive allocation in $\mathcal{E}(\rho)$. Precisely, we assume that ρ is obtained by splitting A into two commodities A_1 and A_2 and we prove that, in equilibrium, agent 3 cannot afford $\frac{1}{4}$ units of goods of type A_1 or A_2, implying that 3 receives a strictly lower utility under ρ. The same strategy shows that if ρ is obtained by splitting B then agent 4 strictly prefers π to ρ.

Assume $t \in \left(0, \frac{1}{2}\right)$ such that $\omega(A_1) = t$ and $\omega(A_2) = \frac{1}{2} - t$. Let p be a competitive price in $\mathcal{E}(\rho)$ normalized so that $p(B) = 1$ and let w be agent 3's wealth at p. We assume that $p(A_1) \leq p(A_2)$ (the other case is treated identically) so that agent 3 demands exactly:

$$\frac{w}{p(A_1)} = \frac{1}{4}\left[t + \frac{p(A_2)}{p(A_1)}\left(\frac{1}{2} - t\right) + \frac{1}{2p(A_1)}\right]$$

units of commodity A_1.

Let us assume by contradiction that $w/p(A_1)$ is greater than $\frac{1}{4}$. There are two possible cases:

- *if $p(A_1) = p(A_2) \leq 1$, then each of the agents 1, 2 and 3 demands $\frac{1}{4}$ units of commodity A_1 or A_2. This creates an excess of demand and thus p cannot be an equilibrium price. On the other hand, if $p(A_1) = p(A_2) > 1$ then $w/p(A_1)$ is strictly less than $\frac{1}{4}$.*

- *If $p(A_1) < p(A_2)$ then agents 1 and 3 demand A_1 instead of A_2. Therefore, $p(A_2) \leq 2$, or no agents would demand A_2. At the same time, it must be that $p(A_1) \geq \frac{1}{2t}$ or agent 1 would demand only A_1, leaving 3 with strictly less than $\frac{1}{4}$ units of A_1. Combining these two inequalities we obtain:*

  $$\frac{w}{p(A_1)} = \frac{1}{4}\left[t + \frac{p(A_2)}{p(A_1)}\left(\frac{1}{2} - t\right) + \frac{1}{2p(A_1)}\right] \leq \frac{1}{4}\left[t + 2t(1-2t) + t\right] = t - t^2$$

  which is strictly smaller than $\frac{1}{4}$ for every $t < \frac{1}{2}$.

The above example is based on refinements of π formed only by 3 intervals. If we allow for richer classifications, then we can find refinements of π that are strictly preferred to π by every agent in the society. As a way of illustration, let ρ be formed by the intervals:

$$A = \left[0, \frac{1}{4} - \varepsilon\right), \quad B = \left[\frac{1}{4} - \varepsilon, \frac{1}{2}\right], \quad C = \left(\frac{1}{2}, \frac{3}{4} - \varepsilon\right], \quad D = \left(\frac{3}{4} - \varepsilon, 1\right]$$

with $\varepsilon \in \left(0, \frac{1}{4}\right)$. For ε sufficiently small, an equilibrium in $\mathcal{E}(\rho)$ is achieved when all commodities have identical prices and each agent consumes the whole of a commodity (1 gets A, 2 gets C, 3 gets B, and 4 gets D). This leaves every agent with a utility strictly larger than the one they received with the allocation (x^i).

The following example refines both Example 4 in the main text and Example 8 above by describing an economy where every refinement of the starting classification gives a strictly lower social welfare. The setup is similar to Example 4, but the set of feasible classifications is curtailed by assuming that the commodity B is an atom, so that some tradable commodities cannot be split into smaller parts.

Example 9. *Let λ be the Lebesgue measure on $\mathcal{I}$ and $\delta_{\{1\}}$ denote the Dirac measure for the singleton $\{1\}$. We consider a society where there are $2n$ agents with identical claims and the measure ω is given by:*

$$\omega(F) = \frac{1}{2}\left(\lambda(F) + \delta_{\{1\}}(F)\right).$$

There are only two types of agents, forming groups of equal size. Agents have linear preferences based on the evaluation measures:

$$\nu_1(F) = \lambda(F) \quad \textit{and} \quad \nu_2(F) = \frac{1}{4}\lambda\left(F \cap \left[0, \frac{1}{2}\right]\right) + \frac{3}{4}\lambda\left(F \cap \left[\frac{1}{2}, 1\right]\right) + \frac{1}{2}\delta_{\{1\}}(F).$$

Intuitively, agents of type 1 value all types of goods identically, while those of type 2 care more about goods in $\left[\frac{1}{2}, 1\right)$ and especially about those labelled with 1.

Let π be the classification formed by the commodities $A = [0, 1)$ and $B = \{1\}$. At the competitive equilibrium, A and B have the same prices, with every agent from group 1 consuming $\frac{1}{2n}$ units of commodity A and every agent from group 2 consuming $\frac{1}{2n}$ units of B.

We prove that, if $\rho \succ \pi$, then every competitive allocation in $\mathcal{E}(\rho)$ assigns a *positive amount of goods of type A to agents in group 2. Because the utility received from goods of type A is higher for agents in group 1, this implies that the sum of agents' utilities in $\mathcal{E}(\rho)$ must be strictly lower than in $\mathcal{E}(\pi)$.*

Suppose by contradiction that there exists a refinement ρ of π and a competitive allocation in $\mathcal{E}(\rho)$ such that agents in group 1 consume all goods of type A and those in group 2 all goods of type B. Because B is an atom, ρ can refine π only by splitting A into smaller intervals and leaving B intact. We write $\rho = \{A_1, \ldots, A_m, B\}$ where $i < j$ implies $s < t$ for all $s \in A_i$ and $t \in A_j$. Because agents from group 1 demand all commodities $A_1, \ldots, A_m$, these must have all equal prices (otherwise agents of group 1 would demand only the cheapest ones). At the same time, A_m must cost strictly more than B, otherwise agents in group 2 would rather demand A_m than B. Hence, the average price of the commodities A_j's is strictly greater than the price of B, implying that each agent in group 2 can demand more than $\frac{1}{2n}$ units of B. This leads to an excess of demand for B, which contradicts the assumption that prices are competitive.

4 Equilibrium without prices

The next two examples illustrate that neither sufficient condition in Theorem 7 can be dropped. A third following example shows that they are not necessary.

Example 10 (A society where ω is non-atomic but no agent has SPC)**.** *There are n agents and ω is the Lebesgue measure. Every agent i has linear preferences with an evaluation measure defined by:*

$$\eta_i(F) = \int_F u_i \, d\omega.$$

for some strictly increasing density u_i, so that no agent exhibits SPC. We claim that no classification based on $k \geq 2$ intervals can support an equilibrium.

Take any classification $\pi = (B_1, \ldots, B_k)$ and let $0 = \theta_0 < \theta_1 < \cdots < \theta_k = 1$ be such that θ_{j-1} and θ_j are the extreme points of the interval B_j. An agent i maximizes the utility $V_i(\pi, x)$ by demanding positive amounts only for the tradable commodities B_j for which the ratio

$$\frac{\eta_i(B_j)}{\omega(B_j)} = \frac{\int_{\theta_{j-1}}^{\theta_j} u_i \, d\omega}{(\theta_j - \theta_{j-1})}$$

is maximized. On the other hand, because u_i is an increasing function, the map $t \mapsto \int_0^t u_i \, d\omega$ is convex and so:

$$\frac{\int_{\theta_{k-1}}^{\theta_k} u_i \, d\omega}{(\theta_k - \theta_{k-1})} > \frac{\int_{\theta_{k-2}}^{\theta_{k-1}} u_i \, d\omega}{(\theta_{k-1} - \theta_{k-2})} > \cdots > \frac{\int_{\theta_0}^{\theta_1} u_i \, d\omega}{(\theta_1 - \theta_0)}.$$

Because every agent demands exclusively the same k-th tradable commodity, there is a positive excess of demand under any classification π with $k \geq 2$. We conclude that no such classification can support an equilibrium.

In Example 10 agents have additive evaluation capacities: their demands are not affected by the width of the intervals in the classification π. This no longer holds if a consumer exhibits SPC, because that consumer is attracted to sufficiently smaller cells.

Example 11 (A society where every agent has SPC but ω is atomic)**.** *Consider an economy where half of the total amount of goods correspond to the point 0 and the other half correspond to 1. Then the measure ω has two atoms and assigns to each $F \subseteq \mathcal{I}$ the value*

$$\omega(F) = \frac{1}{2}\delta_{\{0\}}(F) + \frac{1}{2}\delta_{\{1\}}(F)$$

There are n agents, with linear preferences based on the evaluation measure $\eta_i(F) = \delta_{\{1\}}(F)$; thus, every agent exhibits SPC.

We claim that no classification π based on $k \geq 2$ intervals can support an equilibrium. Given any π, every agent prefers the cell B containing 1 over any other cell and therefore demands only this commodity. This implies a positive excess of demand for B, and the conclusion follows.

Example 11 shows how the presence of large chunks of identical goods can make agents' demands insensitive to changes in the classification. This cannot occur when the measure ω is non-atomic, because the amount of goods labelled with the same $t \in \mathcal{I}$ is negligible.

Example 12 (A society where ω is atomic and no agent has SPC, but equilibrium exists)**.** *There are three agents. The measure ω is defined by*

$$\omega(F) = \lambda\left(F \cap \left[0, \frac{2}{3}\right]\right) + \frac{1}{3}\delta_{\{1\}}(F),$$

where λ denotes the Lebesgue measure. Assume that the three agents have linear preferences based on the evaluation measures:

$$\eta_1(F) = \int_F 2t\,dt, \qquad \eta_2(F) = \eta_3(F) = \int_F 2(1-t)\,dt.$$

Note that ω has the atom $\{1\}$ and that no agent has SPC.

Consider the classification $\pi = \{[0, 2/3], (2/3, 1]\}$. Then agent 1 demands the π-bundle $x^1 = (0, 1)$, while agents 2 and 3 demand the π-bundle $x^2 = x^3 = (1/2, 0)$. Because $\langle \pi, (x^a) \rangle$ is an allocation, we conclude that π supports an equilibrium.

One can relax some assumptions on the model in Subsection 5.2 without compromising the existence result of Theorem 7. We illustrate two possible extensions.

Measure space for the goods' characteristics. We assume that the space of goods' characteristics is a totally ordered set and that commodities are defined as intervals. This can be relaxed to an abstract measure space for the goods, where commodities are defined by measurable subsets. We sketch the main features of this more general approach.

Let $(X, \mathcal{F})$ be a measurable space. We interpret each element t in X as a complete description of a good and each F in $\mathcal{F}$ as a commodity. A non-negative measure ω on $\mathcal{F}$ describes the availability of goods. A *classification of goods* is a partition π of X formed by finitely many sets in $\mathcal{F}$ with positive ω-measure. The definitions for bundles, agents' evaluations and equilibrium are naturally adapted to this more general setup.

Even in this broader setting, there exists a non-trivial classification supporting an equilibrium if ω is non-atomic and at least an agent has SPC. In fact, one can define a family of classifications with similar properties to those formed by intervals of $\mathcal{I}$ and have almost identical proofs. The main intuition is to choose an increasing family of sets, and then mimic a "moving-knife procedure" to define partitions similar to those formed by intervals in $\mathcal{I}$.

Formally, let $\mathcal{C} = \{C_t : t \in \mathcal{I}\} \subseteq \Sigma$ be a monotone chain such that $\omega(C_t) = t$ for all $t \in \mathcal{I}$. Such a chain exists by the non-atomicity of ω. A set J is a *$\mathcal{C}$-interval* if there exists $t < s$ in $\mathcal{I}$ such that $J = C_s \setminus C_t$. Let $\Pi^{\mathcal{C}}_{(\leq k)}$ be the set of classifications formed by at least a number $k \geq 2$ of $\mathcal{C}$-intervals. One may extend the proof of Theorem 7 with respect to $\mathcal{C}$-intervals in X instead of intervals in $\mathcal{I}$.

Note that this more general setting has a much larger class of classifications than $\mathcal{I}$ has. Therefore, although an equilibrium exists, other results may no longer hold. For example, our proof that there exists a Pareto-optimal configuration within the set of competitive configurations cannot be directly extended.

Weaker form of SPC. The assumption that at least an agent has SPC is restrictive, because it requires that there is an agent that will drastically change his choice whenever he is offered a sufficiently concentrated commodity. From a technical viewpoint, however, this assumption is used only to show that the aggregate demand correspondence meets some standard boundary conditions. Therefore, it can be relaxed into a local requirement: if the interval defining a commodity is sufficiently small, then there is at least one consumer who prefers it to all the other commodities. More precisely, consider the following assumption of *distributed SPC*:

> If $\pi^n = (C_1^n, \dots, C_i^n, \dots C_k^n)$ is a sequence of classifications in $\Pi_{(\leq k)}$ and $\omega(C_i^n) \to 0$ as $n \to \infty$, then there exists an agent whose demand for commodity C_i^n goes to infinity as $n \to \infty$.

Under distributed SPC, the proof of Theorem 7 holds unchanged.

Compare the import of SPC versus distributed SPC to appreciate the greater realism of this latter. For the explanatory example in the introduction, SPC requires that there is an agent who, given any classification, might change his choice if he is offered another type of wine using a purer selection of grapes; distributed SPC requires only that, for any classification, there is some agent willing to.

References

[1] C.D. Aliprantis and O. Burkinshaw (1998), *Principles of Real Analysis*, London: Academic Press.