Economists and Mathematics from 1494 to 1969 Beyond the Art of Accounting

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The master economist must possess a rare combination of gifts.
... He must be mathematician, historian, statesman,
philosopher to some degree.
J. M. Keynes [1]

Introduction

The first link between mathematics and economics dates back to calculations of a commercial nature, which merchants had to carry out daily in ancient times. This relationship was later enriched by the financial calculations needed by usurers and bankers: it developed into a *corpus* of applications of arithmetic that is nowadays called accounting. Its practical relevance is at the basis of the importance attributed to the ability of getting sums right, or numeracy, by anyone investing their savings, carrying out a breakdown of tax in an invoice or filling in their tax returns declaration.

During the High Middle Ages, when numeracy was an art limited to few, merchants developed a method for "balancing the accounts" of their businesses and keeping track of income and expenditure. It was in Italy, probably in the second half of the thirteenth century, that double-entry book-keeping was developed: this system is still being used and is still at the heart of the book-keeping activity in any business today. A first description of its methodology was published in Venice in 1494 within a compendium of economics and mathematics by Luca Pacioli (1445 ca.-1514 ca.), *Summa de Arithmetica, Geometria Proportioni et Proportionalitate*. In homage to the merchants' lagoon city that hosted the third convention on "Mathematics and Culture", Venice will be our starting point in telling the story of the evolution of the relationship between mathematics and economics.

Our thesis is simple: as long as the practical needs of commerce or finance were satisfied by the arithmetic collected in book-keeping manuals, the relationship between mathematics and economics was constant but superficial. Merchants needed to keep their accounts tidy and mathematicians possessed the art necessary to do this. The progressive transformation of secular economic precepts into a social science caused the surfacing of both a deeper relationship between economics and mathematics and a new profession – that of economist – entitled to manage it.

Modern economists often have not had the opportunity to use double-entry bookkeeping. However, before becoming economists, they had to study differential calculus, linear algebra, probability theory and statistics. Later, depending on their specialisation, they are required to achieve quite a deep knowledge of those areas of mathematics that they need to build, understand and use quantitative models. Their activity, nowadays, resembles that of an engineer who strives to realise the precepts of an art that has reached the *status* of a science.

Since 1969, a symbol of this status has been the Nobel Prize in Economic Sciences (or, more precisely, the Bank of Sweden Prize in Economic Sciences in memory of Alfred Nobel). Our story of the evolution of the relationship between mathematics and economic sciences starts from the year of the publication of the first work on mathematics and book-keeping (1494) and ends with the international recognition of the figure of the economist as a scientist (1969). During this period of almost five hundred years[1], the relationship between mathematics and economics radically changes.

Mathematics, seen initially as a way to express economic concepts, has increasingly become one of the languages in which economic theory can be formulated and communicated. Having reached the *status* of language, all economists must face it. In the best of cases, it becomes a genuine knowledge tool. More often, though, it is used as a technique to prove the logical coherence of a theory. At times, its use degrades into rhetoric abuse whose only aim is to give a scientific appearance to an argument. In any case, by the time that the Nobel Prize in Economic Sciences starts being awarded, knowledge of mathematics is considered a requisite for this profession.

This article summarises the main phases of the story of this transformation through the voice (or, rather, the pen) of many of its protagonists, by offering a small anthology with comments quoted from their works. Some of the quotes in this article have been inspired by a reading of [2]. Further details on the role of mathematics in the construction of economic theory can be found in [3] and [4].

Mathematics as a source of examples

The mathematical competences of the first economists are probably limited to the arithmetic necessary to book-keeping and accountancy. Since economics deals with prices and values, theoretical discussions cannot avoid the use of numbers altogether. But this use is mostly an example to persuade the reader that “the accounts balance”, that is that what has been stated derives from the internal logic of economics.

In fact, the reflection on economics is not yet a science. Researchers are trying to uncover the “natural behavioural laws” of economic systems and arithmetic is contributing decisive help in identifying the economic phenomena that need to be

[1] The three-decade period between 1969 to 1999 is dealt with in the article “Who said that a mathematician cannot win the Nobel Prize?”, also published in this volume.

explained. In *his Essay on the Nature of Commerce*, written around 1720 but published in 1755, Richard Cantillon (1680-1734) illustrates how the same amount of work carried out by a man can correspond to a different buying power:

> If, for example, one man earn an ounce of silver every day by his work, and another in the same place earn only half an ounce, one can conclude that the first has as much again of the produce of the land to dispose of as the second.

In his *An Inquiry into the Nature and Causes of the Wealth of Nations* (1776), Adam Smith (1723-1790), unanimously considered the father of economic science, does not use proportions – but rather multiplication and division – to clarify how specialisation can increase a worker's productivity:

> a workman not educated to this business […] certainly could not make twenty [pins a day]. […But] where ten men only were employed, and where some of them consequently performed two or three distinct operations […] they could […] make among them upwards of forty-eight thousand pins in a day. Each person, therefore, […] might be considered as making four thousand eight hundred pins in a day

Only a few years later, Thomas Malthus (1766-1834) introduces arithmetic and geometric progressions in his *An Essay on Population* (1789) to explain the risk that population growth might drastically reduce the quality of life:

> Taking the population of the world at any number, a thousand millions, for instance, the human species would increase in the ratio of -- 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, etc. and subsistence as -- 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, etc. In two centuries and a quarter, the population would be to the means of subsistence as 512 to 10.

Cantillon claims that a different salary can correspond to the same quantity of work. Smith explains that a different organization of work can increase a worker's productivity. Malthus shows that the rate of population increase can drastically diverge from the rate of development of means of subsistence. Each of these propositions can be expressed without using mathematics. However, using an arithmetic example causes the argument to stand out with greater clarity. The "accounts balance", because what is claimed can be verified numerically, which reassures the reader as to the coherence of the discourse.

Mathematics as a language

Slowly – and mostly unwittingly – the use of mathematics as source of clarifying examples will lead economists to introduce a more explicitly mathematical language in their writing. In this process of cultural enrichment, three trends emerge.

On the one hand, there is a sincere effort to formulate the laws of economics rigorously, by introducing precise definitions and mathematically-proven propositions. An example of this is the way in which, in his *Recherches sur les Principes*

Mathématiques de la Théorie des Richesses (1838), Antoine A. Cournot (1801–1877) proves the existence of a price that maximizes revenue:

> Since the [demand function] *F(p)* is continuous, the function *pF(p)*, representing the value of the yearly sold quantity, must also be continuous. […] Since *pF(p)* initially rises and then dips in *p*, there must exist a value of p that maximises this function. This value is given by the equation $F(p) + pF'(p) = o$.

The existence of this price – optimum price because it leads to the highest possible revenue – justifies economists' efforts to supply precepts that help businessmen determine the best price at which to sell their products. The proof is a simple but genuine application of mathematics to economics. Not for nothing, Cournot is often considered the father of mathematical economics. However, scholars like Cournot remain rare until 1870 and Cournot himself will not write about economics after 1838.

More common, instead, is the tendency to introduce a more mathematical language as a natural complement to a search for rigour, which however is not identified with mathematical rules. The main communication means used by scholars in economics remains mainly non-mathematical, with occasional intrusion of mathematical terminology.

As a representative of this prevailing trend, we can consider the manual *Principles of Political Economy* (1848, 1st ed.; 1871, 7th ed.) by John Stuart Mill (1806–1873) – the reference text in the formation of economists until at least the second half of the nineteenth century:

> The equation of international demand […] can be concisely stated as follows. The produce of a country exchanges for the produce of other countries, at such values as are required in order that the whole of her exports may exactly pay for the whole of her imports.

Mill states that the exchange rate of a currency must be such that the value of imports is equal to the value of exports. Expressed as "equation of international demand", the same proposition can be given in a mathematical form. Thus, mathematics can be used as a language to discuss economics.

One hundred years later, we find full awareness of this in an anecdote about physicist Gibbs reported by Paul A. Samuelson (born in 1915 and winner of Nobel Prize in economics in 1970) in a passionate defence of the importance of mathematics to economic theory [5]:

> The great Willard Gibbs was supposed ever to have made [one single speech] before the Yale Faculty. […While professors] were hotly arguing the question of required subjects: Should certain students be required to take languages or mathematics? Each man had his opinion of the relative worth of these disparate subjects. Finally Gibbs, who was not a loquacious man, got up and made a four-word speech: "Mathematics is a language".

The spreading among economists of the awareness that mathematics can be a language with which one can communicate economic theory and build applications is one of the important causes of all the successive transformations in the relationship between mathematics and economics. For simplicity, we will gather them in three categories.

For most, following Mill, the assimilation of the axiomatic method and demonstration techniques borrowed from mathematics will provide the basis for the transformation of a collection of vague and contradictory economic concepts into a systematic and logically coherent *corpus*. In Samuelson's words [5]:

> The problems of economic theory – such as the incidence of taxation, the effects of devaluation – are by their nature quantitative questions. […] When we tackle them by words, we are solving the same equations as when we write out those equations. […] Where the really big mistakes are is in the formulation of premises. […] One of the advantages of the mathematical medium – or, strictly speaking, of the mathematician's customary canons of exposition of proof, whether in words or symbols – is that we are forced to lay our cards on the table so that all can see our premises.

Some great scholars will follow Cournot's example. Going beyond the use of mathematics as a mere technique, they will turn it into a genuine knowledge tool indispensable for the advancement of economic theory. Again, Samuelson writes [5]:

> Euler's Theorem is absolutely basic to the simplest neoclassical theory of imputation. Yet without mathematics, you simply cannot give a rigorous proof of Euler's theorem.

Finally, for others, the paraphrase of mathematical language will only serve to attribute more authority to their theses. Leaving out the examples of Smith and Malthus, since these are simple and understandable by anyone, in this case mathematics is not used for cultural enrichment, but to obscure laymen's understanding of economic laws. John Maynard Keynes (1883–1946), possibly the greatest economist ever, warns against this degenerative temptation in an anecdote told by his friend Roy F. Harrod in his biography *The Life of John Maynard Keynes* (1951):

> When I asked him in 1922 how much mathematics it was needful for an economist to know, he replied that Johnson, in his article in the Economic Journal, had carried the application of mathematical analysis to economic theory about as far as it was likely to be useful to carry it.

In order to better evaluate this claim, it should be pointed out that the article by Johnson that Keynes refers to was published in 1913: it used differential calculus and determinants. It is, therefore, a mathematical background similar to that which (in 1922!) is expected from an engineer.

The three following sections will show how, in its relationship with economics, mathematics has been seen and used as a technique, a knowledge tool and a rhetorical tool.

Mathematics as a Technique

Let us first of all consider the role of mathematics in the development of economic analysis techniques. An example will help us in understanding the terms of reference. Let us read again the passage by Chantillon – quoted earlier – on the problem of establishing which "natural law" gives the first man's working day a higher value than the second man's working day. Presumably, the work carried out by the first man is paid more because it is more productive than the work carried out by the second man. Let us now formulate two hypotheses whose concatenation can explain the phenomenon at hand: 1) a higher productivity creates a higher value; 2) a higher value results in a higher salary. How can we turn these hypotheses into knowledge?

One could proceed by empirical induction, by observing whether a higher productivity corresponds to a higher value and whether a higher value corresponds to a higher salary. However, since the value of a working day is not a measurable quantity, we can only ascertain whether a higher productivity results in a higher salary. The result of this experiment is not conclusive: a positive correlation between productivity and salary is a necessary but not sufficient condition for the existence of the two positive correlations we hypothesised. Vice versa, if we find a negative correlation, we cannot determine which of the two hypotheses is correct.

In general, this is a typical problem encountered in all social sciences: the consequence is that it is difficult to devise conclusive experiments for even the simplest propositions. Because this prevents us from employing empirical induction, many scholars have been forced to proceed deductively. If we define the concepts of salary, value and productivity appropriately, we can write a theorem and show that better productivity creates higher value, even if not necessarily a higher salary.

The first scholar to consciously stress the importance of the deductive method in mathematics as a basis technique for economic science was William S. Jevons (1835–1882) in his *opus magnum* on *The Theory of Political Economy* (1871):

> Economy, if it is to be a science at all, must be a mathematical science.

The characteristics of his method are evident in his letter dated 1st June 1860 to his brother Herbert:

> I obtain from the mathematical principles all the chief laws at which political economists have previously arrived, only arranged in a series of definitions, axioms, and theories almost as rigorous and connected as if they were so many geometrical problems.

Jevons was followed by a radical methodological change – known as marginalist revolution – which revolutionised economics. Marginalists' introduction of the deductive method is at the basis of the neoclassical theory, which even today constitutes the paradigm of economic science. For many years, the historians of economic thought have been debating the relationship between the introduction of the marginalist method and progress in economic science. Here we will simply observe that it has two concomitant effects.

The first effect is that of "mathematicising" part of the economic discourse, opening the discipline to the contributions of other scientists, particularly physicists and engineers. They, in turn, concentrate on and study the problems that best fit the deductive method. Economic science gains in depth in some areas, but at the same time, risks losing track of other – equally important – areas. In fact, the second effect is that of pushing into the background all historical and institutional aspects that are difficult to treat "mathematically".

The most characteristic example of the first effect is the introduction of a utility function to describe consumers' behaviour. As Tjalling Koopmans (1910–1985, Nobel Prize in economics in 1975) explains in his *Three Essays on the State of Economic Science* (1957):

> A utility function of a consumer looks quite similar to a potential function in the theory of gravitation.

Engineers familiar with rational mechanics – among these Italian Antonelli, Pareto and Boninsegni – or physicists used to the principle of conservation of energy only need to transpose their knowledge into the economic field to generate a myriad of new results and enlightening analogies. The economist who best represents this process is, again, Samuelson. In his article *How "Foundations" Came to Be* (1998), he writes:

> I was vaccinated early to understand that economics and physics could share the same formal mathematical theorems (Euler's theorem on homogeneous functions, Weierstrass's theorems on constrainted maxima, Jacobi determinant identities underlying LeChatelier reactions, etc.), while still not resting on the same empirical foundations and certainties.

Between 1870 and 1930, the success of these exchanges is accompanied by a methodology for the use of mathematics in economic research. This methodology is quite close to what was in use in rational mechanics at the end of the nineteenth century. Not for nothing, the greatest success of neoclassical economics is the theory of general economic equilibrium conceived by Leon Walras (1834–1910) and consolidated by Vilfredo Pareto (1848–1923) who sees economics as a system of contrasting forces in search of an equilibrium.

Nowadays, more modern mathematical methods have been introduced: yet, this formulation survives in the most mathematically accessible – and therefore more widespread – formulations of neoclassical theory. This has two main effects. The first is that economists often continue to adopt those hypotheses that can be more easily taken back to this analogy, instead of using the most realistic ones. In fact, in this respect, very little progress has been made since October 1901, when Henri Poincaré wrote Walras a letter criticising his axioms of economic behaviour:

> You regard men as infinitely selfish and infinitely farsighted. The first hypothesis may perhaps be admitted in a first approximation, the second may call for some reservation.

The second effect is the little attention that economists usually dedicate to alternative paradigms (such as evolutionary theories from biology) or to the most recent theory in physics (such as relativity or quantum theories). Yet, in some cases, mathematical theories have been suggested by economic problems and later re-discovered in physics. But, in these cases, more than a technique, mathematics is being used as a knowledge tool and this is the topic of the next section.

Mathematics as a Knowledge Tool

The most successful applications of mathematics to economics have extolled its role as a knowledge tool. Numerous economic theories have been set out and perfected thanks to the systematic and enlightened use of mathematics. A thorough review of such theories can be found in [6].

Among these applications, we will mention seven: a) the theory of general economic equilibrium, based on a mathematics similar to that of rational mechanics and later on the methods of differential and algebraic topology; b) the theory of rational expectations, founded on statistical inference and on dynamic programming; c) game theory, which has even generated its *ad hoc* mathematics; d) economics of uncertainty, based on the same theory as game theory as well as on probability calculus; e) the theory of social choice, founded on algebraic methodology; f) mathematical finance, which has enriched the theory of continuous time stochastic processes; g) the theory of optimal resource allocation, from which linear programming – and, more generally, operational research – developed.

Some of the main authors of these theories have received the Nobel Prize in economics. Further details can be found in the article "Who said that a mathematician cannot win the Nobel Prize?", published in this volume, which also deals with Debreu and economic equilibrium, Nash and game theory, Arrow and social choice and Kantorovich and optimal allocation.

We will mention three mathematicians who died before 1969: Ramsey, von Neumann and Bachelier. At least in the case of the first two, we are certain that the only reason they did not receive a Nobel Prize in economics was its late institution. In fact, in a brief autobiographical piece written in 1990, Robert T. Solow (born in 1924 and Nobel Prize in economics in 1987), claims that the three non-economists who have been most important to economics have been Ramsey, von Neumann and the mathematical statistician Harold Hotelling (1895–1973).

The first author is Frank P. Ramsey (1903–1930), who died at a very young age, but authored absolutely original contributions on three different problems: providing a general criteria for decision-making under conditions of uncertainty (1926, published posthumously in 1931); determining the best income taxation system (1927) and establishing the best way to accumulate national savings (1928). The latter work introduced the use of calculus of variation in economics, bringing to economists' attention the necessity to introduce techniques to solve dynamic optimisation problems.

The second author is John von Neumann (1901–1957): versatile genius, with sporadic interest in economics, he nevertheless provided absolutely original contributions to this discipline. In fact, he found the first general solution to purely antagonistic games between two people (1928); provided the first model of economic growth with conditions for the permanence in time of the equilibrium (1937); together with Oskar Morgenstern (1902–1977), he placed into a system framework some previous intuitions by founding game theory and decision theory (1944).

The third author, still unknown to most economists even today, is Louis Bachelier (1870–1946). In his doctoral thesis (1900) on speculation problems, he represented price movements of financial activities through random walks, anticipating by a few years Einstein's Brownian motion (1905) and by seventy years the use of martingales in the mathematical representation of an efficient market.

The contribution of mathematics to economics, however, is not limited to its role as a knowledge tool with respect to theories. As said earlier, starting from 1870, the marginalist revolution consciously transforms the use of mathematics from a language into an analysis technique. In this process, economic theory is more and more characterised by the systematic use of the axiomatic formalism typical of the deductive method. The limitation of this approach is that it allows the development of theories of general importance, but does not provide predictive models bound in space and time.

Starting from 1930, under the pressure of statisticians, a second – and parallel – process of formalisation concerning empirical observations affirms itself. To understand the world and foresee what is going to happen, a quantitative science is needed which can manipulate statistical data, from which it needs to derive descriptive and predictive models, whose validity is based on formal but empirical criteria. In a sense, it is the revenge of the inductive method over the deductive method. Next to mathematical economics, which reduces economic phenomena to theorems, econometrics is born, with its aim to measure economic phenomena.

The birth certificate of this new discipline is the foundation of the Econometric Society, "*an international society for the advancement of economic theory in its relation to statistics and mathematics*" by Irving Fisher (1895–1973) and Ragnar Frisch (1895–1973, Nobel Prize in economics in 1969). Here is how the latter describes econometrics in the article *On a Problem in Pure Economics* (1926):

> Half way between mathematics, statistics and economics, we find a new discipline that, for want of a better name, we can call *econometrics*. The purpose of econometrics is to subject the abstract laws of economic theory or "pure" economics to experimental and numerical tests, so as to turn pure economics, as far as possible, into a science in a strict sense.

After some understandable initial difficulty, the Econometric Society, also thanks to a series of favourable financial circumstances, has an unexpected success and econometrics establishes itself rapidly, distinguishing itself from traditional economics. Frisch is usually credited with the invention of the terms "microeconomics", "macroeconomics" and "econometrics" which even today still designate the three main subjects in the first year economics undergraduate curriculum.

Before closing this section on the successes of mathematical economics, it may be useful to compare Frisch's full-of-hope words with an anecdote told by Keynes in his *Essays in Biography* (1933):

> Planck […], the famous originator of Quantum Theory, once remarked to me that in early life he had thought of studying economics, but had found it too difficult! […] Planck could easily master the whole corpus of mathematical economics in a few days. […] But the amalgam of logic and intuition and the wide knowledge of facts, most of which are not precise, which is required for economic interpretation in its highest form is, quite truly, overwhelmingly difficult for those whose gift mainly consists in the power to imagine and pursue to their furthest points the implications and prior conditions of comparatively simple facts which are known with a high degree of precision.

Even if econometrics and mathematical economics have contributed to transforming economics into a science, their task is not over. Economics and society are constantly changing and their genuine interpretation requires yet more mathematical instruments and knowledge.

Mathematics as a Rhetorical Tool

A consequence of the successes in the application of mathematical methods and instruments to economics is the spreading of the mathematical language as a rhetorical tool. Banal propositions, when duly made up to look like a theorem, can look like scientific arguments (particularly to lay people). In these cases, unfortunately, mathematics contribute neither to the analysis nor to the development of economic science, but is reduced to a tool used to dress up and confer greater dignity to studies which – evidently – have little merit in themselves.

The thesis that economics suffers from an excessive "mathematisation" is often based on this widespread rhetorical artifice. Very many economists, and even some Nobel Prize winners such as M. Allais and W. Leontief, have attacked – sometimes violently – the practice of passing mathematical theorems as good economics.

One of the most well-known criticisms to the use of mathematics as an economic analysis tool was made by Alfred Marshall (1842–1924) in a letter to Bowley dated 27th November 1906:

> A good mathematical theorem dealing with economic hypotheses [is] very unlikely to be good economics: […] the rules --- (1) Use mathematics as a shorthand language, rather than as an engine of inquiry. (2) Keep to them till you are done. (3) Translate into English. (4) Then illustrate by examples that are important in real life. (5) Burn the mathematics. (6) If you can't succeed in (4), burn (3). This last I did often.

Coherently, in his *Principles of Economics* (1890, 1st ed.; 1920, 8th ed.), the most widely-used economics manual across the nineteenth and twentieth century, Marshall relegated his formal system to the appendix. But, as his pupil Keynes explains in his *Essays in Biography* (1933), he did so to avoid giving the impression that mathematics provides answers to real-life problems just by itself.

A necessary condition for a fertile application of mathematics to economics is that it must contribute to the understanding of important phenomena. Otherwise, it is better to be silent. The risk that mathematical formalism can take over the substance of problems, on the other hand, is the same risk described in general by John von Neumann in [7]:

> As a mathematical discipline travels far from its empirical source […], it is beset with very grave dangers. It becomes more and more purely aestheticizing, more and more purely l'art pour l'art. This need not be bad, if the field is surrounded by correlated subjects, or if the discipline is under the influence of men with exceptionally well-developed taste. But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches, and that the discipline will become a disorganised mass of details and complexities.

Mathematics as a Professional Requisite

The versatility of the roles that mathematics carries out in economics – such as technique, analysis and rhetorical tool – implies that it is to be considered by all means a professional requisite indispensable to the modern economist.

Here is Samuelson's advice to a youth with modest mathematical background who wishes to study economic analysis in depth [5]:

> It happens to be empirically true that if you examine the training and background of all the past great economic theorists, a surprisingly high percentage had, or acquired, at least an intermediate mathematical training. […] Moreover, without mathematics you run grave psychological risks. As you grow older, you are sure to resent the method increasingly. Either you will get an inferiority complex […] or you will get an inferiority complex and become aggressive about your dislike of it. […] The danger is almost greater that you will overrate the method's power for good or evil.

A good mathematical awareness must provide economists with a capacity to tell a good theory from a bad one and prevent them from being taken in by rhetorical sirens. After all, mathematics is a tool that should be judged on the basis of the use that is made of it. He goes on to advise as follows:

> Mathematics is neither a necessary nor a sufficient condition for a fruitful career in economic theory. It can be a help. It can certainly be a hindrance, since it is only too easy to convert a good literary economist into a mediocre mathematical economist.

There is no mathematical *via regia* to economics. But it is a fact that the importance of mathematics in professional economic communication is undisputed. For instance, a researcher of G.J. Stigler (1911–1991, Nobel Prize in economics in 1982) and other two collaborators noted that, taking into account the whole of the five main economic reviews, the percentage of articles making use of differential calculus or other advanced techniques has risen from 2% in 1932–33 to 31% in 1952–53 and from 46% in 1962–63 to 56% in 1989–90 [8].

Perhaps more surprisingly, another study by T. Morgan [9] notices how in 1982–83 the "mathematical models without empirical data" were 42% in the *American Economic Review* (a reference review for economists), 18% in the *American Political Science Review* (reference review for political scientists), 1% in the *American Sociological Review* (reference review for sociologists), 0% in the *Journal of the American Chemical Society* (reference review for chemists) and only 12% in the *Physical Review* (reference review for physicists).

Apparently, therefore, there is (by far!) more mathematics in economics than in any of the other social sciences and even than in more traditional scientific disciplines. It is therefore all the more important for economists to have a solid mathematical background, so as to avoid suffering from any inferiority complexes and to be able to distinguish good from bad economics autonomously. A last reason is not to betray that quarter of mathematical nobility that, as quoted in the epigraph, Keynes attributes to a true economist.

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