Multidisciplinary Robust Optimization for Ship Design

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ABSTRACT

1 INTRODUCTION

Design optimization formulations and techniques are intended for supporting the designer in the decision making process, relying on a rigorous mathematical framework, able to give the "best" solution to the design problem at hand. Over the years, optimization has been playing an increasingly important role in engineering. Advanced modeling and algorithms in optimization constitute now an essential part in the design and in the operations of complex aerospace (Hicks and Henne, 1978; Sobieszczanski-Sobieski and Haftka, 1997; Alexandrov and Lewis, 2002; Willcox and Wakayama, 2003; Morino et. al., 2006; Iemma and Diez, 2006) and automotive (Baumal et. al., 1998; Kodiyalam and Sobieszczanski-Sobieski, 2001) applications, when, for example, it is by all means important to reduce costs and shorten time of development. In the design of large and complex systems, the use of efficient optimization tools leads to better product quality and improved functionality (Mohammadi et. al., 2001). The success of design optimization has attracted the naval community, so that the recent years have seen progress in optimization for ships too (Ray et. al., 1995; Peri and Campana, 2003; Parsons and Scott, 2004; Pinto et al., 2004; Peri and Campana, 2005; Campana et al., 2007, 2009; Papanikolaou, 2009).

Generally speaking, the task of designing a ship (as well as an aerial or ground vehicle) possibly requires that the engineering team considers a host of multidisciplinary design goals and requirements. Multidisciplinary Design Optimization (MDO) classically refers to the quest for the best solution with respect to optimality criteria and constraints, whose definition involves a number of disciplines mutually coupled. Therefore, MDO encompasses the interaction of different discipline-systems, formally joined together and inter-connected in a multidisciplinary framework, which leads to a multidisciplinary equilibrium.

In this context, design engineers increasingly rely on computer simulations to develop new designs and to assess their models. However, even if most simulation codes are deterministic, in practice systems' design should be permeated with uncertainty. On this guideline, the most straightforward example in the naval hydrodynamics context is offered by any existing ship, that must perform under a variety of operating conditions (e.g. different, stochastic environmental conditions). The general question is now: "how can the results of computer simulations be properly exploited in the framework of design optimization, when the overall context is affected by uncertainty ?" Moreover "how can deterministic analysis be integrated in an *ad hoc* formulation that includes uncertainty ? How can it be used to get designs that are relatively insensible to stochastic variations of the external inputs and of the variables?". The latter questions stress one of the major issues arising in the optimization of a (ship) design: *the perspective from which the optimization task has to be formulated and performed.* Indeed, one may argue that a "tight" deterministic optimization leads to specialized solutions that are often inadequate to face the "real-life" world, which is instead characterized by a high level of uncertainty. In other words, specialized optimization procedures which include only deterministic parameters are often unable to model the overall problem and, consequently, are unable to provide adequate solutions to it. In this respect Marczyk (2000) states that, in a deterministic engineering context, *optimization* is the synonymous of *specialization* and, consequently, the opposite of *robustness*. The perspective we try to give in the present work has the aim of broadening the standard-optimization-problem framing, leading to a formulation in which *optimality* is redefined in terms of *robustness*, rather than *specialization*. To the aim of clarifying the latter perspective, it may be useful to summarize the following statements:

- Design optimization is always about answering a question, i.e. assisting the designer in the decision making process.

- As a consequence and necessarily, before going through

the optimization procedure, special attention has to be paid to the formulation of the problem. In the context of design optimization, incomplete or coarse models very often yield inadequate answers.

- In this work we try to re-formulate the optimization perspective by looking at the design problem from a broader standpoint. Moreover, we bring the uncertainty related to ship design, manufacturing and operations, into the decision problem.

- The formulation of the question (we try to answer to, using optimization) relies on optimal statistical decision theory and, specifically, on Bayes criteria, defining a rigorous mathematical framework in which the "robust" decision making process is embedded.

In general, in any engineering system, the uncertainty is due to variations of design, operating or environmental conditions. The uncertainty is also related to the evaluation of the relevant functions, due to inaccuracy in modeling or computing. Using ideas from statistical decision theory, and specifically Bayes criteria (De Groot, 1970), the problem of robust decision making in design can be formulated as an optimization problem (Robust Design Optimization, RDO). In the framework of Bayes theory, we assume that the original "deterministic" design goal is the minimization of a general loss function (e.g., the performance). The expectation of this loss, with respect to the uncertainty involved in the process, is defined as the risk associated to the stochastic scenario assumed. In this context, the final goal is that of minimizing the risk, looking for the so-called Bayesian solution to the problem. In other words, once a probabilistic scenario is assumed, the optimization task reduces to the minimization of a related loss expectation.

The difficulty with exploiting this framework is both *theoretical* and *computational*. The latter is due to the fact that the evaluation of the loss expectation involves the numerical integration of expensive simulation outputs, with respect to uncertain quantities. The former can also be easily understood: in a more standard MDO formulation (as well as in standard *deterministic* numerical optimization), all the relevant variables, parameters and functions are defined from a deterministic viewpoint and, apparently, the optimization process does not involve any kind of stochastic variation. The resulting optimal solutions are therefore likely specialized for the specific scenario assumed. Nevertheless, the performances of the final design may significantly drop in *off-design* conditions, when the deterministic assumptions used no longer hold. In this context, we look for a *robust* solution to the MDO problem, i.e., a solution able to represent a good performance on average, in the whole range of variations of the probabilistic scenario. The effects of properly considering the uncertainty, mainly consist in a loss

in specialization of the system and a gain in robustness. The MDO problem, re-formulated to take into account uncertainty, becomes a Multidisciplinary Robust Design Optimization (MRDO) problem. The aim of the present work is to analyze the combined effects of considering several disciplines and uncertainty in ship design problems, developing a MRDO procedure that utilizes efficient methods for uncertainty analysis and encompasses the features of the MDO framework. Theory and applications of MDO subject to uncertainty may be found in, e.g., Agarwal et al. (2004); Du and Chen (2000a,b, 2002); Giassi et al. (2004); Mavris et al. (1999); Smith and Mahadevan (2005), and Sues et al. (1995).

It may be noted that, in naval applications (Diez and Peri, 2009, 2010a,b), as well as in aeronautical problems (Padula et al., 2006), the usage and environmental conditions may be considered as "intrinsic" stochastic functions, whose expected values and standard deviations can neither be influenced by the designer nor by the manufacturer. Conversely, uncertainties related to design variables and functions' evaluation reflect the current state of and technology, and theoretically may be reduced by improving modeling, computing and manufacturing processes. Thus, in this work, the MRDO problem is formulated taking into account the stochastic variation of the operating conditions. The (joint) probability density function of the operating scenario are taken as a design requirement and the expectation of the relevant merit factors is assessed during the optimization task. For solving the minimization problem, a Particle Swarm Optimization (PSO) algorithm, in the form proposed by Campana et al. (2009), is used.

The application studied in this work consists in the optimization of a keel fin of a sailing yacht. The keel fin provides the side force able to contrast the wind, allowing the yacht to travel along directions not aligned with the wind itself. The keel sustains an heavy ballast bulb, and the bending moment generated by this configuration, as well as by the hydrodynamic loads, generate an elastic displacement, which cannot be ignored in the computation of the hydrodynamic performances. As a consequence, a fully coupled hydroelastic problem is considered. The solution of the deterministic configuration has been illustrated in Campana et. al (2006). In this paper, a MRDO problem will be defined and solved, considering a probabilistic sailing scenario, in terms of cruise speed, heel and yaw angles.

The paper is organized as follows. The next section presents the general context of optimization problems affected by uncertainty. Then, in Section 3, Bayes theory is exploited to formulate the present problem for RDO. The general framework of MDO is presented in Section 4, whereas the "robust" extension of MDO to MRDO is given in Section 5. Finally, the numerical results are pre-

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sented in Section 6 and the concluding remarks are given in Section 7.

2 DESIGN OPTIMIZATION SUBJECT TO UNCERTAINTY

In this Section, an overview of an optimization models for a problem affected by uncertainty is presented. In this context, the designer concern is that of finding an optimal configuration able to keep a good performance in a wide range of variation of some uncertain parameters. In order to achieve such an optimal solution, an optimality criterion, based on robustness of the final design, has to be defined. We remark that here, the term "robust" is always associated with the uncertainty of parameters. Therefore, attention to robustness always involves care to handle some kind of uncertainty. A number of authors in the literature give different meanings to robustness depending on the application, and different kind of uncertainties are addressed. The interested reader is referred to Beyer and Sendhoff (2007); Park et al. (2006); Zang et al. (2005).

In order to define the context of the present work, the following standard-deterministic optimization problem is considered:

$$\underset{\mathbf{x} \in X}{\text{minimize}} \quad f(\mathbf{x}, \mathbf{y}), \quad \text{given } \mathbf{y} = \hat{\mathbf{y}} \in Y$$
$$\text{subject to} \quad g_n(\mathbf{x}, \hat{\mathbf{y}}) \leq 0, \quad n = 1, ..., N \quad (1)$$
$$h_m(\mathbf{x}, \hat{\mathbf{y}}) = 0, \quad m = 1, ..., M$$

where $\mathbf{x} \in \mathbf{X} \subseteq \mathbb{R}^{\mathbf{k}}$ is the design variables vector (which represents the *designer choice*), $\hat{\mathbf{y}} \in Y$ is the design parameters vector (which collects those *parameters independent of the designer choice*, e.g., environmental or usage conditions defining the operating point), and $f, g_n, h_m : \mathbb{R}^k \to \mathbb{R}$, are respectively the optimization objective and the inequality and equality constraint functions. While handling the above problem, the following uncertainties may occur – the interested reader is also referred to Diez and Peri (2010a).

a) Uncertain design variable vector. When translating the designer choice into the "real-life" world, the design variables may be affected by uncertainties due to manufacturing tolerances or actuator precision. Assume a specific designer choice $\mathbf{x}^*$ and define as $\boldsymbol{u} \in U$ the error or tolerance related to this choice.[1] We may assume $\boldsymbol{u}$ as a stochastic process with probability density function $p(\boldsymbol{u})$; by definition it is $\int_U p(\boldsymbol{u}) d\boldsymbol{u} = 1$. The

expected value of $\mathbf{x}^*$ is, therefore,

$$\overline{\mathbf{x}^*} := \mu(\mathbf{x}^* + \boldsymbol{u}) = \int_U (\mathbf{x}^* + \boldsymbol{u}) \, p(\boldsymbol{u}) \, d\boldsymbol{u}. \quad (2)$$

It is clear that, if the stochastic process $\boldsymbol{u}$ has zero expectation, i.e.

$$\overline{\boldsymbol{u}} := \mu(\boldsymbol{u}) = \int_U \boldsymbol{u} \, p(\boldsymbol{u}) \, d\boldsymbol{u} = 0 \quad (3)$$

we obtain $\overline{\mathbf{x}^*} = \mathbf{x}^*$. It may be noted however that, in general, the probability density function $p(\boldsymbol{u})$ depends on the specific designer choice $\mathbf{x}^*$.

b) Uncertain environmental and usage conditions. In "real-life" applications, environmental and operational parameters may differ from the design conditions $\hat{\mathbf{y}}$ (see Problem (1)). The design parameters vector may be assumed as a stochastic process with probability density function $p(\mathbf{y})$ and expected value or mean

$$\overline{\mathbf{y}} := \mu(\mathbf{y}) = \int_Y \mathbf{y} \, p(\mathbf{y}) \, d\mathbf{y}. \quad (4)$$

Note that, in this formulation, the uncertainty on the usage conditions is not related to the definition of a specific design point. Environmental and operating conditions are treated as "intrinsic" stochastic processes in the whole domain of variation Y, and the designer is not requested to pick a specific design point in the usage parameters space. For this reason, we do not define an "error" in the definition of the usage conditions, preferring the present approach which identifies the environmental and operational parameters in terms of their probabilistic distributions in the whole domain of variation.

c) Uncertain evaluation of the functions of interest. The evaluation of the functions of interest (objective and constraints) may by affected by uncertainty due to inaccuracy in modeling or computing. Collect objective and constraints in a vector $\mathbf{f} := [f, g_1, ..., g_N, h_1, ..., h_M]^T$, and assume that the assessment of $\mathbf{f}$ for a specific "deterministic" design point, $\mathbf{f}^* := \mathbf{f}(\mathbf{x}^*, \hat{\mathbf{y}})$, is affected by a stochastic error $\boldsymbol{w} \in W$. Accordingly, the expected value of $\mathbf{f}^*$ is

$$\overline{\mathbf{f}^*} := \mu(\mathbf{f}^* + \boldsymbol{w}) = \int_W (\mathbf{f}^* + \boldsymbol{w}) \, p(\boldsymbol{w}) \, d\boldsymbol{w}. \quad (5)$$

Note that, in general, the probability density function of $\boldsymbol{w}$, i.e. $p(\boldsymbol{w})$, depends on $\mathbf{f}^*$ and, therefore, on the design point $(\mathbf{x}^*, \hat{\mathbf{y}})$.

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[1] The symbol * is used in the present formulation to denote a specific designer choice. While $\mathbf{x}$ represents all the possible choices in X, $\mathbf{x}^*$ defines a specific one.

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Combining the above uncertainties, we may define the expected value of **f** as

$$\overline{\mathbf{f}} := \mu(\mathbf{f}) = \iiint\limits_{UYW} \big[\mathbf{f}(\mathbf{x}^* + \boldsymbol{u}, \mathbf{y}) + \boldsymbol{w}\big]$$
$$p(\boldsymbol{u}, \mathbf{y}, \boldsymbol{w}) \, d\boldsymbol{u} \, d\mathbf{y} \, d\boldsymbol{w} \qquad (6)$$

where $p(\boldsymbol{u}, \mathbf{y}, \boldsymbol{w})$ is the joint probability density function associated to $\boldsymbol{u}, \mathbf{y}, \boldsymbol{w}$. It is clear that $\overline{\mathbf{f}} = \overline{\mathbf{f}}(\mathbf{x}^*)$; in other words, the expectation of **f** is a function of the only designer choice. Moreover, the variance of **f** with respect to the variation of $\boldsymbol{u}, \mathbf{y}, \boldsymbol{w}$ is

$$V(\mathbf{f}) := \sigma^2(\mathbf{f}) = \iiint\limits_{UYW} \big\{ \big[\mathbf{f}(\mathbf{x}^* + \boldsymbol{u}, \mathbf{y}) + \boldsymbol{w}\big] -$$
$$\overline{\mathbf{f}}(\mathbf{x}^*)\big\}^2 p(\boldsymbol{u}, \mathbf{y}, \boldsymbol{w}) \, d\boldsymbol{u} \, d\mathbf{y} \, d\boldsymbol{w} \quad (7)$$

resulting, again, in a function of the designer choice variables. The evaluation of the integrals in Equations (6) and (7) is often referred as Uncertainty Quantification (UQ).

With respect to the uncertainties outlined above, different approaches may be followed for the re-definition of the optimization problem. Specifically, the optimization task may be defined in terms of:

- minimization of the variance, or of the standard deviation, $\sigma := \sqrt{V}$, of f: this leads to a robust design in a strict sense – e.g., Taguchi methods (Taguchi, 1986);

- minimization of the expectation of f: if f represents a general loss in the performance, then the expected value of f can be seen as a risk – Bayesian approach as in statistical decision theory (De Groot, 1970; Trosset et al., 2003) – see next section;

- minimization of f in the worst possible case; this is the most conservative approach – "minmax" approach, see again Trosset et al. (2003);

- assessing probabilistic constraints in the minimization of the objective function (Tu et al., 1999; Sues et. al, 2001; Du and Chen, 2000b; Agarwal, 2004; Agarwal and Renaud, 2004).

With respect to the previous approaches, different definitions may be found in the literature – the interested reader is referred, again, to Beyer and Sendhoff (2007):

- *Robust design* (RD): process of defining the robust design in the strict sense (e.g., Taguchi methods). The attention in this case is mainly on variance or standard deviation.

- *Robust optimization or Robust design optimization* (RDO): optimization process considering uncertainties in the evaluation of the objective function; expected value, variance, worst case, etc. may be taken into account.

- *Reliability-based design optimization* (RBDO): the attention is focused on the statistical feasibility of the design (i.e., on the constraints). The constraints are treated as probabilistic inequalities and give a statistical feasible region.

While RD and RDO are mainly focused on expectation and variance of a cost function (Zang et al., 2005), the RBDO concentrates on the handling the uncertainty of the constraints (Tu et al., 1999; Sues et. al, 2001; Du and Chen, 2000b; Agarwal, 2004; Agarwal and Renaud, 2004). The latter are treated as probabilistic inequalities (Nocedal and Wright, 1999) and the n-th deterministic constraint of the type $g_n(\mathbf{x}, \mathbf{y}) \leq 0$ is treated using the general probabilistic statement

$$P_S := P\big[g_n(\mathbf{x}, \mathbf{y}) \leq 0\big] \geq P_0 \qquad (8)$$

where P_S is the probability of success, $P[A]$ denotes the probability of the event A and P_0 is a given target probability. Note that the probability of failure, P_F, equals $(1 - P_S)$. In the following, the constraints of the optimization problem will be defined in the worst possible case, choosing a conservative approach ($P_0 = 1$). The issues connected to the probabilistic handling of the constraints are beyond the scope of the present work, and will be not further addressed here.

3 DECISION MAKING UNDER UNCERTAIN OPERATING CONDITIONS: ROBUST DESIGN OPTIMIZATION THROUGH BAYES THEORY

In this section, specific attention is paid to the uncertainty related to the environmental and operating conditions. As mentioned, in the context of naval applications, environmental and operating conditions may be considered as "intrinsic" stochastic functions, whose expected values and standard deviations can neither be influenced by the designer nor by the manufacturer. For this reason, assessing probabilistic operating conditions, may be interpreted as a relevant step towards a more comprehensive design optimization, bringing into focus "real-life" applications.

In the following, the formulation for robust design optimization subject to uncertain environmental and operating conditions is presented. To this aim, assume that the optimization objective in Problem 1 is associated to a general loss (like, for instance, the performance loss with respect to a given target). Under the hypothesis of uncertain environmental and operating conditions, we may

refer to $f(\mathbf{x}, \mathbf{y})$ as the loss associated to the designer choice $\mathbf{x}$, when the condition $\mathbf{y}$ occurs. Therefore, the expectation of the loss f, evaluated through the integral of Equation (6) (limited to uncertain parameters $\mathbf{y}$, thus referring to uncertainty of type $\mathbf{b}$ only), may be defined as the risk associated to the decision $\mathbf{x}$ under the distribution $p(\mathbf{y})$ (De Groot, 1970). It follows that the designer should choose, if possible, a decision $\mathbf{x}$ which minimizes the risk (expected loss). Specifically, if we consider the Bayes risk, i.e. the lower bound of the expected loss for all the possible choices in X,

$$\rho := \inf_{\mathbf{x} \in X} \bar{f} = \inf_{\mathbf{x} \in X} \mu(f) \qquad (9)$$

we look for the Bayes decision of the problem, considering the distribution $p(\mathbf{y})$, i.e. the decision for which the risk equals the Bayes risk ρ. Therefore, the optimal designer choice is that which minimizes the expected loss of the system performances with respect to the stochastic variation of the environmental and operating conditions collected in $\mathbf{y}$. It may be noted that in the present context, *the design specifications are no longer given in terms of a single operating design point, but in terms of probability density function of the operating scenario.*

It may be noted that the Bayesian approach to the decision problem may be enriched by considering, as a second objective function, the standard deviation of f. The latter extension improves the robustness of the final design to the operating conditions variation.

The Bayesian approach to the designer decision problem (eventually enriched by considering the standard deviation of f) may be formulated as follows:

$$
\begin{aligned}
\underset{\mathbf{x} \in X}{\text{minimize}} \quad & f_1(\mathbf{x}) := \mu\big[f(\mathbf{x}, \mathbf{y})\big] \\
\text{and} \quad & f_2(\mathbf{x}) := \sigma\big[f(\mathbf{x}, \mathbf{y})\big] \qquad (10) \\
\text{subject to} \quad & \sup_{\mathbf{y} \in Y} \big\{g_n(\mathbf{x}, \mathbf{y})\big\} \le 0, \quad n = 1, ..., N \\
& \mu\big[h_m(\mathbf{x}, \mathbf{y})\big] = 0, \quad m = 1, ..., M
\end{aligned}
$$

where, $\mu(f)$ and $\sigma(f)$ are given, by definition, through the UQ:

$$\mu\big[f(\mathbf{x}, \mathbf{y})\big] := \int_Y f(\mathbf{x}, \mathbf{y}) p(\mathbf{y}) d\mathbf{y} = \bar{f}(x) \quad (11)$$

$$\sigma^2\big[f(\mathbf{x}, \mathbf{y})\big] := \int_Y [f(\mathbf{x}, \mathbf{y}) - \bar{f}]^2 p(\mathbf{y}) d\mathbf{y} = \sigma^2(x). \quad (12)$$

4 MULTIDISCIPLINARY DESIGN OPTIMIZATION

The basic elements of the MDO problem can be easily summarized. We assume that each discipline is based on a disciplinary analysis (from simple algebraic formulas to complex PDEs) that may be schematically depicted as an input-output relation: In the context of design op-

Figure 1: *Discipline analysis Δ_i of the i-th discipline.*

timization subject to uncertain operating conditions, the input of each discipline is a set of *deterministic* design variables, $\mathbf{x} := \{\mathbf{x}_i^T, \mathbf{x}_S^T\}^T$, *uncertain* operating parameters, $\mathbf{y} := \{\mathbf{y}_i^T, \mathbf{y}_S^T\}^T$, and a set of parameters supplied by other disciplines, $\{\mathbf{a}_j\}_{j \ne i} := \{\mathbf{a}_1^T, \ldots, \mathbf{a}_{i-1}^T, \mathbf{a}_{i+1}^T, \ldots, \mathbf{a}_n^T\}$; the analysis produces a set of outputs, $\mathbf{a}_i$. The system-level design variables, $\mathbf{x}_S$ and the system level operating parameters $\mathbf{y}_S$, are those shared by all the disciplines. The disciplinary design variables, $\mathbf{x}_i$ are assumed local to Δ_i as well as the disciplinary operating parameters $\mathbf{y}_i$.

The disciplinary analysis has the functional form $\mathbf{a}_i = A_i(\mathbf{x}_i, \mathbf{y}_i, \mathbf{x}_S, \mathbf{y}_S, \mathbf{a}_1, ..., \mathbf{a}_{i-1}, \mathbf{a}_{i+1}, ..., \mathbf{a}_n)$, where A_i is assumed to be independent of $A_j, \forall j \ne i$. In the context of the MDO problem, the coupled Multidisciplinary Analysis (MDA) system reflects the physical requirement that a solution simultaneously satisfies all the disciplinary analyses. The multidisciplinary analysis system, in the explicit form, is therefore given by the simultaneous system of equations:

$$
\begin{cases}
\mathbf{a}_1 = A_1(\mathbf{x}_S, \mathbf{y}_S, \mathbf{x}_1, \mathbf{y}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n) \\
\mathbf{a}_2 = A_2(\mathbf{x}_S, \mathbf{y}_S, \mathbf{x}_2, \mathbf{y}_2, \mathbf{a}_1, \mathbf{a}_3, \ldots, \mathbf{a}_n) \\
\vdots \\
\mathbf{a}_n = A_n(\mathbf{x}_S, \mathbf{y}_S, \mathbf{x}_n, \mathbf{y}_n, \mathbf{a}_1, \ldots, \mathbf{a}_{n-1})
\end{cases}
\qquad (13)
$$

Solving the fully coupled system Equation (13) leads to a full multidisciplinary analysis. The solution is in this case a consistent solution that satisfies all the disciplines.

Up to now we have just looked for a multidisciplinary equilibrium among the disciplines. The most natural MDO problem formulation is to impose an optimizer over the MDA Equation (13) and find the optimal solution with respect to the *deterministic* designer choice $\mathbf{x}$. Figure 2 presents, at a glance, a deterministic two-disciplines MDO procedure, where no uncertainties are considered during the optimization.

Figure 2: *MDO procedure.*

5 MULTIDISCIPLINARY ROBUST DESIGN OPTIMIZATION

The extension of the above procedure, to take into account the application of Bayes principle to an uncertain operating scenario, involves the integration of the objective f over the uncertain parameters domain. It may be noted that the uncertainties propagate in the MDA framework. With respect to each discipline involved, the uncertainty related to the definition of the input variables and parameters may be referred as *external*, whereas the source of uncertainty related to the analysis tool itself (e.g., inaccuracy in computing) is addressed as *internal* (Du and Chen, 2002). As a result, the final multidisciplinary equilibrium is affected by uncertainty.

As clearly appears, the solution of the Multidisciplinary Robust Design Optimization problem represents an expensive task, due to the fact that the integrals of Equations (11) and (12) apply to a function supplied by the multidisciplinary equilibrium of Equation (13), for every value of the uncertain parameters $\mathbf{y}$. It is worth noting that the standard deterministic MDO scheme involves coupling an optimization algorithm (*optimizer*, see Figure 2) with the MDA framework. Taking into account the uncertainty in that context (thus formulating the MRDO problem) requires the insertion between the optimization algorithm and the MDA, of an UQ scheme (as summarized in Figure 3).

Figure 3: *MRDO procedure.*

6 NUMERICAL RESULTS

6.1 Uncertainty Quantification (UQ)

6.2 Multidisciplinary Robust Design Optimization (MRDO)

7 CONCLUDING REMARKS

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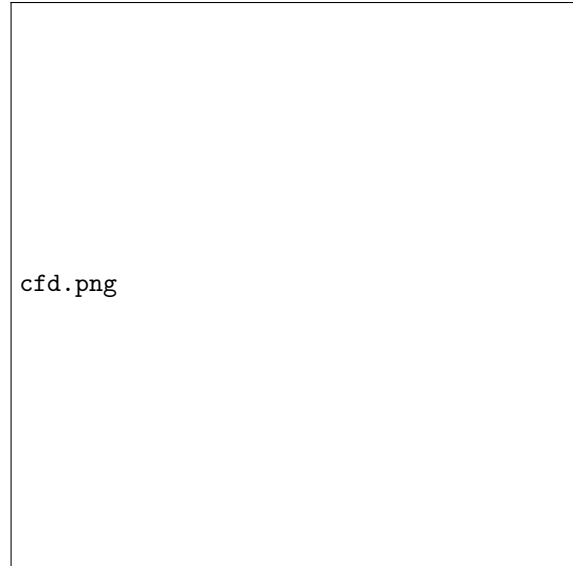Willcox, K., Wakayama, S., 2003. Simultaneous optimization of a multiple-aircraft family. Journal of Aircraft, 40 (4).

Figure 4: *Hydrodynamics (Δ_1) - CFD solution for the pressure field.*

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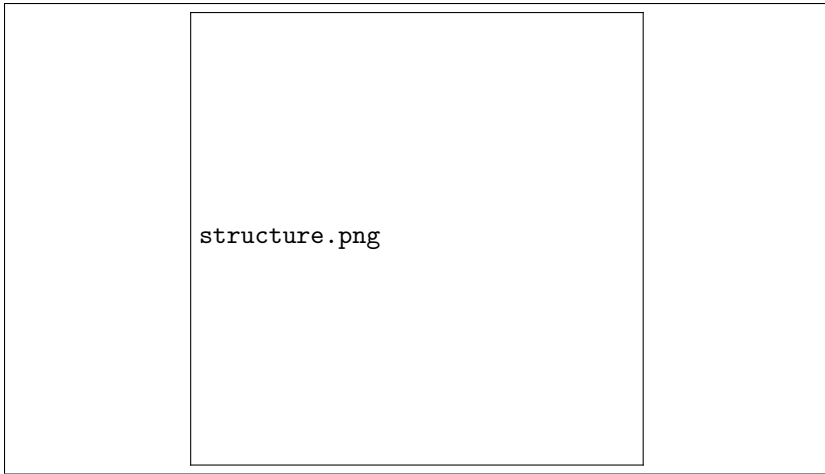
Figure 5: *Structural analysis (Δ_2) - FEM solution for the elastic displacements of the fin.*