Application of derivative-free multi-objective algorithms to reliability-based robust design optimization of a high-speed catamaran in real ocean environment[1]

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ABSTRACT: A reliability-based robust design optimization (RBRDO) for ship hulls is presented. A real ocean environment is considered, including stochastic sea state and speed. The optimization problem has two objectives: (a) the reduction of the expected value of the total resistance in waves and (b) the increase of the ship operability (reliability). Analysis tools include a URANS solver, uncertainty quantification methods and metamodels, developed and validated in earlier research. The design space is defined by an orthogonal four-dimensional representation of shape modifications, based on the Karhunen-Loève expansion of free-form deformations of the original hull. The objective of the present paper is the assessment of deterministic derivative-free multi-objective optimization algorithms for the solution of the RBRDO problem, with focus on multi-objective extensions of the deterministic particle swarm optimization (DPSO) algorithm. Three evaluation metrics provide the assessment of the proximity of the solutions to a reference Pareto front and their wideness.

1 INTRODUCTION

Simulation-based design (SBD) optimization is an essential part of the design process for complex engineering systems. In shape design, geometry modification tools are coupled with simulation codes and optimization algorithms in order to solve the design problem. The process is often affected by different sources of uncertainties (such as operational, environmental, geometrical or numerical) and require uncertainty quantification methods and reliability-based robust design optimization (RBRDO) formulations to identify optimal solutions, in the stochastic sense. The numerical solution of the RBRDO problem is usually computationally very costly (especially if high-fidelity simulations are used) and may be achieved by means of metamodels, coupled with effective optimization algorithms.

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Herein, a RBRDO for ship hulls is solved, for real ocean environment with stochastic sea state and speed. The problem is taken from earlier research (Diez et al., 2013) and is formulated as a multi-objective optimization problem aimed at (a) the reduction of the expected value of the resistance in waves and (b) the increase of the ship operability (which coincides herein with the reliability of the design, with respect to a set of given contraints). The design space is defined by an orthogonal four-dimensional representation of shape modifications, based on the Karhunen-Loève expansion of free-form deformations of the original hull (Chen et al., 2014). The optimization relies on URANS simulations with a stochastic radial basis function metamodel (Volpi et al., 2014) and pertains to the hull-form design of a 100m *Delft* catamaran, sailing in head waves in the North Pacific ocean. For details on the problem formulation and the geometry modification technique, the interested reader is referred to Diez et al. (2013) and Chen et al. (2014).

The objective of the present work is the assessment of deterministic derivative-free multi-objective optimization algorithms for the solution of the RBRDO problem.

The focus is on multi-objective extensions of the deterministic particle swarm optimization (DPSO) algorithm (e.g., Serani et al. 2014). Three approaches for multi-objective deterministic PSO (MODPSO) include generalizations of the single-objective algorithm by: (a) distance from personal and social Pareto fronts, (b) personal aggregated objective and distance from social Pareto front, and (c) vector evaluated particle swarm optimization (VEPSO). Three performance metrics are used, providing the assessment of the proximity of the solutions to the reference Pareto front along with their wideness. The algorithms are evaluated by 66 test functions from literature, and then applied to the catamaran RBRDO problem, varying the number of analysis-tool calls (evaluation budget).

The final presentation will also include a comparison with multi-objective derivative-free (MODFO) algorithms. Specifically, a MODFO method based on a line search-based approach to approximate the local Pareto front will be included as well as a MODFO method encompassing a new globalization technique based on a suitable modification of the well-known DIRECT algorithm.

2 MULTI-OBJECTIVE EXTENSIONS OF DETERMINISTIC PSO

The following single-objective deterministic PSO (DPSO) iteration (see, e.g., Serani et al. 2014) is used in the current work for extension to multi-objective optimization:

$$\begin{aligned} v_i^{t+1} &= \chi \left[v_i^t + c_1 \left(p_i - x_i^t \right) + c_2 \left(g - x_i^t \right) \right] \\ x_i^{t+1} &= x_i^t + v_i^{t+1} \end{aligned} \tag{1}$$

where x_i^t and v_i^t are the (vector-valued) position and velocity of the particle i ($i = 1, ..., N_p$) at iteration t, χ is a damping factor, c_1 and c_2 are coefficients controlling the personal and social behavior of the particles, p_i is the best position ever visited by the i-th particle, whereas g is the best position ever visited by all the particles.

When the number of objective functions, N_{of}, is greater than one, the definition of personal best position, p_i, and global best position, g, should reflect the multi-objective nature of the problem. The DPSO iteration is rewritten as

$$\begin{aligned} v_i^{t+1} &= \chi \left[v_i^t + c_1 \left(p_i - x_i^t \right) + c_2 \left(g_i - x_i^t \right) \right] \\ x_i^{t+1} &= x_i^t + v_i^{t+1} \end{aligned} \tag{2}$$

where the personal best position p_i takes into account multiple objectives, as well as the global term g_i, which is based on the knowledge shared by all the particles and may vary from particle to particle.

2.1 *Pareto-front based MODPSO (*$\mathrm{P}_p\mathrm{P}_g$*)*

The idea behind this variant of MODPSO is that of generalizing the single-objective DPSO, in the Pareto-optimality sense (Diez et al., 2013). Specifically, p_i and g_i are defined as follows:

- p_i is the closest point to the i-th particle of the personal Pareto front of all positions ever visited by the i-th particle;
- g_i is the closest point to the i-th particle of the global Pareto front of all positions ever visited by all the particles.

Distances are evaluated in the design variables space.

2.2 *Pareto-front and aggregate-objective-function based MODPSO (*$\mathrm{AOF}_p\mathrm{P}_g$*)*

This variant of DPSO has been presented in Campana and Pinto (2005) and makes use of an aggregate objective function for the personal term. Accordingly:

- p_i is the personal optimum of all positions ever visited by the i-th particle, with respect to the aggregate objective function, $f_{\mathrm{AOF}} = \sum_{j=1}^k w_j f_j$, where $w_j = 1/N_{of}$;
- g_i is the closest point to the i-th particle of the global Pareto front of all positions ever visited by all the particles.

Table 1: Coefficient sets

Reference	χ	c_1	c_2
Shi and Eberhart (1998)	0.729	2.050	2.050
Trelea (2003)	0.600	1.700	1.700
Clerc (2006)	0.721	1.655	1.655
Campana and Pinto (2005)	1.000	0.400	1.300
Diez et al. (2013)	0.990	0.330	0.660

2.3 *Vector evaluated DPSO (VEPSO)*

This MODPSO variant uses a number of sub-swarms equal to the number of objective functions (Parsopoulos et al., 2004):

- p_i is the personal optimum of all positions ever visited by the i-th particle of the j-th swarm, with respect to the j-th objective function;
- g_i is the global optimum of all positions ever visited by the all the particles of the k-th swarm ($k \neq j$), with respect to the k-th objective function.

If $N_{of} > 2$, the exchange of information among sub-swarms follows a ring connection.

2.4 *Implementation*

The PSO coefficient sets are taken from literature and included in Tab. 1 (see, e.g. Serani et al. 2014).

The swarm size is set to $N_p = 2^n N_{dv} N_{of}$ with $n \in \mathbb{N}\,[1,6]$ and N_{dv} number of design variables.

The swarm initialization is based on the Hammersley distribution (Wong et al., 1997), which is applied respectively to (a) the whole domain, (b) the domain and its boundaries in even amount, and (c) the domain boundaries only. The initial particles location is combined with null $(v_0 = 0)$ and non-null initial velocity $(v_0 \neq 0)$.

Finally, a semi-elastic wall type approach is used for the box constraints. For details, see Serani et al. (2014).

3 EVALUATION METRICS

Generational Distance (GD) and Inverse Generational Distance (IGD) (see, e.g, Cabrera and Coello 2010) are chosen as performance indicators. An overall performance metric is given, as a Generational Merit Factor (GMF), combining GD and IGS. Specifically,

$$\mathrm{GD} = \frac{\sqrt{\Sigma_{i=1}^{Q} d_i^2}}{Q} \tag{3}$$

where d_i is the distance between the i-th point ($i = 1, ..., Q$) of the Pareto front found and the reference Pareto front;

$$\mathrm{IGD} = \frac{\sqrt{\Sigma_{j=1}^{P} d_j^2}}{P} \tag{4}$$

Table 2: Occurrence of number of variables N_{dv} and objective functions N_{of} values for the test functions

N_{dv}	Occurrence
2	42
3	4
4	4
7	1
8	15

N_{of}	Occurrence
2	43
3	23

where d_j is the distance between the j-th point ($i = 1, ..., P$) of the reference Pareto front and the Pareto front found; finally,

$$\mathrm{GMF} = \sqrt{\frac{\mathrm{GD}^2 + \mathrm{IGD}^2}{2}} \tag{5}$$

Distances d are evaluated in the objective functions space, suitably normalized between minimum and maximum reference values.

4 APPROACH OF ANALYSIS AND NUMERICAL RESULTS

The evaluation metrics GD, IGD and GMF are evaluated as a function of the number of objective-function calls (N_{feval}). The reference Pareto front is defined as the set of non-dominated solutions among all optimizations (obtained by varying the algorithms' parameters), with a number of function evaluations equal to $N_{feval} = 2,000 N_{dv} N_{of}$.

MODPSO algorithms are assessed using 66 test functions taken from Hwang and Masud (1979); Kursawe (1991); Fonseca and Fleming (1998); Cheng and Li (1999); Deb (1999); Jin et al. (2001); Deb et al. (2002); Okabe et al. (2004); Huband et al. (2005, 2006) and Lovison (2010). The number of design variables N_{dv} ranges from two to eight, whereas the number of objective functions N_{of} ranges from two to three. The frequency of occurrence of N_{dv} and N_{of} values in the test functions set is shown in Tab. 2.

Figure 1 shows the average GD, IGD and GMF values obtained by the MODPSO algorithms, over all test functions, coefficient sets, swarm sizes and initializations. $\mathrm{P}_p\mathrm{P}_g$ and $\mathrm{AOF}_p\mathrm{P}_g$ have similar performance, and are more effective that VEPSO. $\mathrm{P}_p\mathrm{P}_g$ has the best performance overall. It may be noted how, on average, GD is found larger than IGD. This is due to non-dominated solutions, found by the MODPSO algorithms, which are still significantly far from the reference Pareto front. Figures 2, 3, and 4 show the relative variance of GD, IGD and GMF, retained by each of the MODPSO parameters (coefficient set, swarm size, initialization), for $\mathrm{P}_p\mathrm{P}_g$, $\mathrm{AOF}_p\mathrm{P}_g$ and VEPSO respectively. All parameters affect significantly the algorithms' performance, and therefore deserve a careful investigation. The best performance overall is given by $\mathrm{P}_p\mathrm{P}_g$ with the coefficient set from Clerc (2006), a swarm size equal to $32 N_{dv} N_{of}$ and initialization of particles over the whole domain with null velocity. The best performance for AOF$_p$P$_g$ is provided by the coefficient set by Clerc (2006), a swarm size equal to $64N_{dv}N_{of}$ and initialization of particles over the whole domain with null velocity. Finally, the best-performing VEPSO is given by the coefficient set by Diez et al. (2013), a swarm size equal to $64N_{dv}N_{of}$, with particles initialized over the domain and the boundaries with non-null velocity.

Figure 5 shows the average GD, IGD and GMF values obtained by the MODPSO algorithms for the catamaran RBRDO problem. GD, IGD and GMF have similar trends. As for the test functions, the choice of the algorithm is found a significant issue. P$_p$P$_g$ and AOF$_p$P$_g$ have very similar performances and are more effective than VEPSO. Figures 6, 7, and 8 show the relative variance of GD, IGD and GMF, retained by each of the MODPSO parameters (coefficient set, swarm size, initialization), for P$_p$P$_g$, AOF$_p$P$_g$ and VEPSO respectively. The best implementation for P$_p$P$_g$ and VEPSO is found using the coefficient set by Diez et al. (2013), with $64N_{of}N_{dv}$ particles for P$_p$P$_g$ and $32N_{of}N_{dv}$ particles for VEPSO, initialized over the domain and the boundaries with null and non null velocity, respectively; the best-performing implementation for AOF$_p$P$_g$ is given by the coefficients by Shi and Eberhart (1998), with $16N_{of}N_{dv}$ particles initially distributed over the domain and the boundaries, with null velocity. The best-performing implementation overall is provided by P$_p$P$_g$. Finally, Fig. 9 shows the best Pareto fronts, with $N_{feval} = 2,000N_{dv}N_{of}$, for each of the algorithms, with comparison with the reference.

5 CONCLUSIONS

A parametric analysis of three MODPSO variants' performance has been given, varying the coefficient set, the swarm size and the initialization of the particles. The algorithms are extension to multi-objective problems of the single-objective DPSO. Three evaluation metrics have been used, namely the generational distance, the inverse generational distance and an overall generational merit factor. Results have been shown for 66 test functions and for a metamodel-based RBRDO of a high-speed catamaran in real ocean environment.

The choice of the algorithm has been found the most significant issue in order for the MODPSO to be effective and efficient. Coefficient set, swarm size and particles initialization also affect significantly the optimization performance (at least for the Pareto-front based algoritms). Overall, P$_p$P$_g$ is found the most effective for both the test functions and the catamaran RBRDO.

Comparison with MODFO methods will be also given in the final presentation, based on a line search-based approach to approximate the local Pareto front and on a new globalization technique derived from a suitable modification of the well-known DIRECT algorithm.

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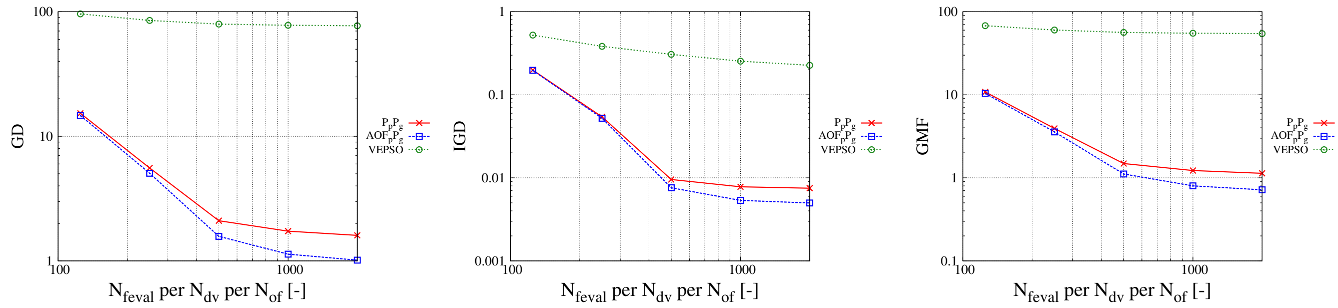

Figure 1: Test functions: average GD, IGD and GMF (from left to right respectively), conditional to the algorithm used

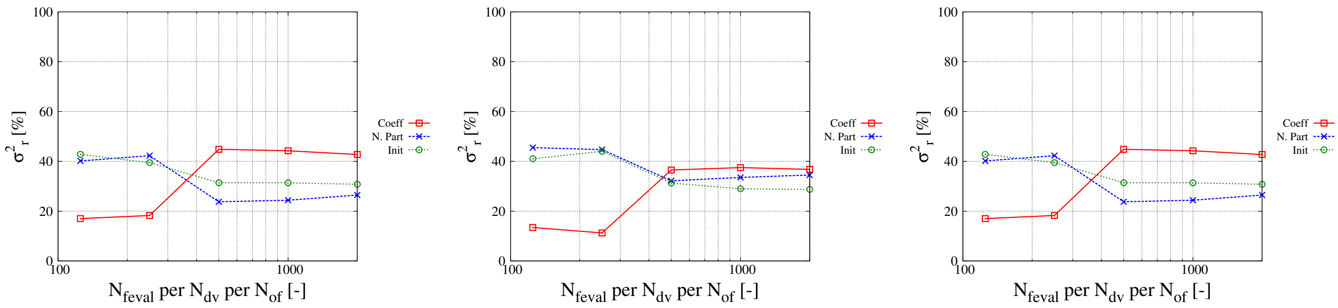

Figure 2: Test functions: relative variance of GD, IGD and GMF (from left to right respectively) for P_pP_g algorithm

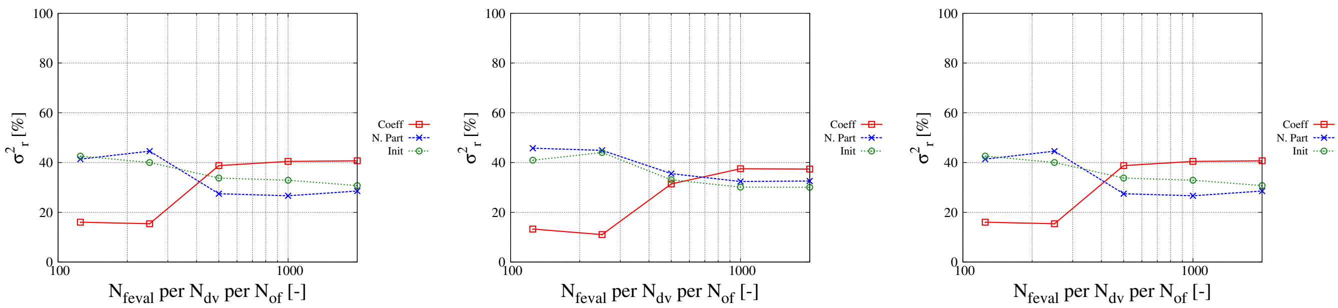

Figure 3: Test functions: relative variance of GD, IGD and GMF (from left to right respectively) for AOF_pP_g algorithm

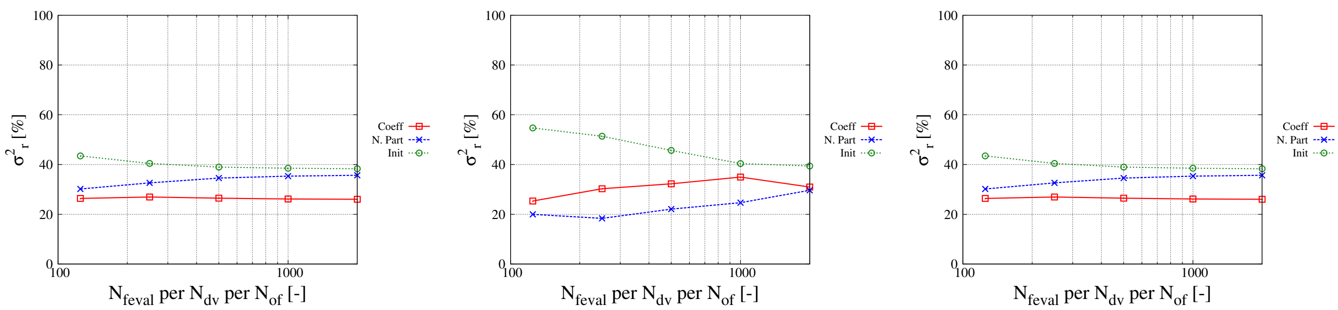

Figure 4: Test functions: relative variance of GD, IGD and GMF (from left to right respectively) for VEPSO algorithm

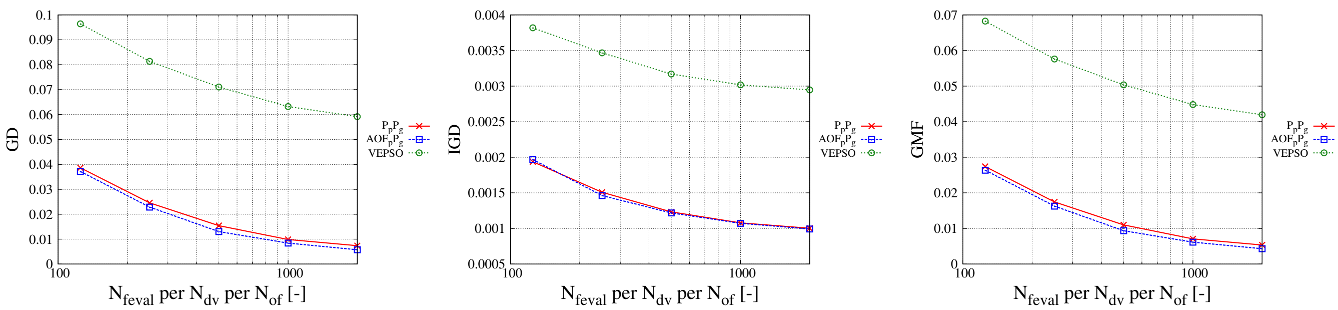

Figure 5: Catamaran RBRDO: average GD, IGD and GMF (from left to right respectively), conditional to the algorithm used

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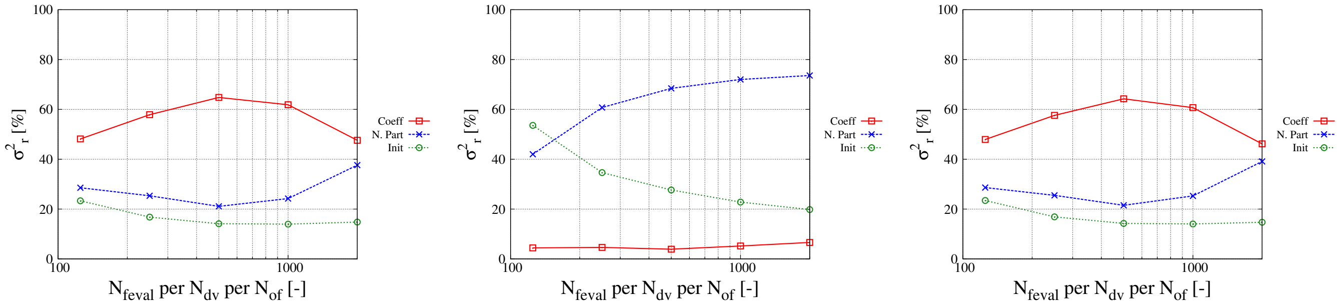

Figure 6: Catamaran RBRDO: relative variance of GD, IGD and GMF (from left to right respectively) for P_pP_g algorithm

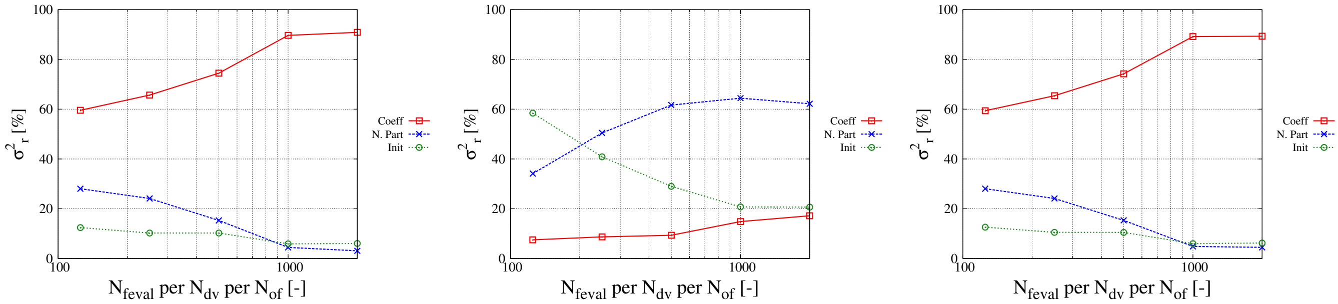

Figure 7: Catamaran RBRDO: relative variance of GD, IGD and GMF (from left to right respectively) for AOF_pP_g algorithm

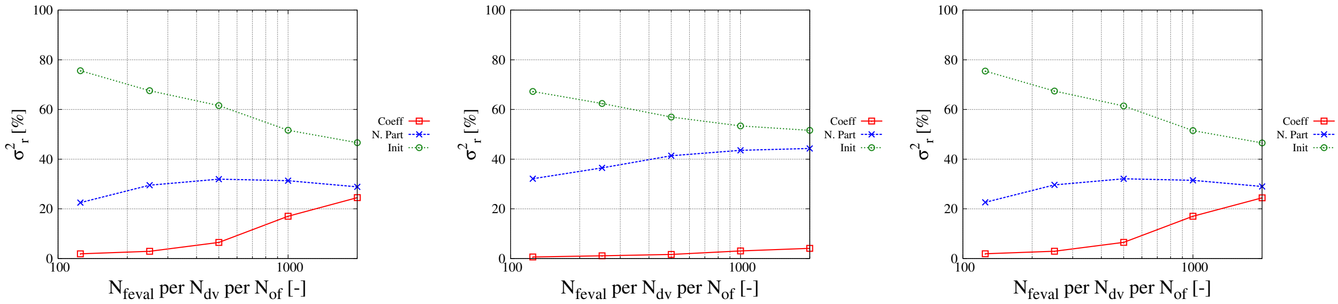

Figure 8: Catamaran RBRDO: relative variance of GD, IGD and GMF (from left to right respectively) for VEPSO algorithm

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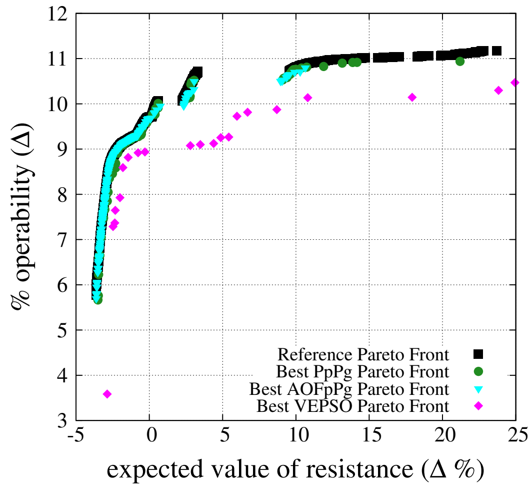

Figure 9: Comparison of Pareto fronts